

## Golden Ratio

### Big Ideas

#### Unit of Instruction

Discovering and using the Golden Ratio ( $\phi$ ) in nature, art, and geometry.

#### Geometry Concept

- Discover the pervasiveness of the Golden Ratio in nature and art
- Calculate the Golden Ratio from measurements
- Compass and straightedge construction of the Golden Ratio—in pentagram, rectangles, and spirals

#### Rationale

In measuring select objects, students will be able to discover mathematics themselves as they calculate  $\phi$ . The pentagon construction allows students to apply their compass construct skills and then discover the golden ratio. The golden spiral, as seen throughout nature, applies the use of the compass and allows students to see the connection to the golden rectangles. Creating the collage allows students to integrate their constructions, and newly acquired knowledge as they make connections to the world of nature.

#### Idaho Content Standards

- G.1.1.1 Understand the meanings of real numbers.
- G.4.1.1 Analyze properties and determine attributes of two- and three-dimensional objects.
- G.4.1.2 Explore congruence and similarity among classes to two-dimensional objects and solve problems involving them.
- G.4.4.1 Draw and construct representations of two-dimensional geometric objects using a variety of tools.
- 9-12.VA.3.1.1 Select and apply media, techniques, and processes effectively and with artistic intention.
- 09-12.VA.3.2.4 Select and utilize visual, spatial, and temporal concepts to enhance meaning in artwork.

<b>Background</b>	
<b>Vocabulary</b>	
Archimedes spiral	Pentagram
Fibonacci numbers	Phi
Geometric spiral	Proportion
Golden ratio	Ratio
Logarithmic spiral	Value (lightness and darkness in art)
Mean/Extreme	
<b>Math Instruction (pre- or post-project)</b>	
<ul style="list-style-type: none"><li>• Calculate and define: ratio and proportion</li><li>• Review measurement with ruler and basic compass construction (swinging arc, bisecting segments, measuring segments, perpendicular bisector)</li></ul>	

## Driving Question

### Project Objective

- Students will measure images of objects in nature, art and geometry to discover the Golden Ratio.
- Students will construct with compass and straightedge:
  - Regular pentagon
  - Pentagram
  - Golden Rectangle
  - Golden Spiral
- Students will calculate the value of phi using measured ratios.

### Extension

- Students will create a collage using images that reflect the Golden Ratio and the Golden Spiral, emphasizing proportion and value contrast.

### Questions to be Answered

- How do you construct a regular pentagon and pentagram using just a compass and straightedge?
- What is phi?
- What is the Golden Ratio?
- How pervasive is phi?
- How do you construct a spiral using Golden Rectangles?
- What are Fibonacci numbers?
- How are Fibonacci numbers related to phi?

### Extension

- What is a collage?
- How do you make a collage?
- What is proportion?
- What is value?

## Materials

### Materials Required

- Good compasses
- Centimeter rulers
- Pencils
- Measuring worksheets (see page 8)
- Calculators
- Graph paper (see page 22)

### Extension (collage)

- Tissue paper
- Paper cups
- Brushes
- 8 ply substrate (illustration board, poster board, scrape mat board, or corrugated cardboard)
- Copies of included images (see pages 9-17)
- Scissors
- Glue

### Reference Materials

- Livio, Mario. *The Golden Ratio*. New York: Broadway Books, 2003.
- Runion, Garth. *The Golden Section*. Palo Alto, CA: Dale Seymour Publications, 1990.
- Additional illustrations of Fibonacci numbers and Golden ratio.

## Lesson Outline

### Description of Activity

- Discover phi (1.618033...) by measuring images included in the back of this lesson (45 minutes)
- Construct spiral using compass and straightedge (45-50 minutes)
- Compass construction of pentagon and pentagram(50 minutes) Collage using images and compass construction(50 minutes)

### Extension

- Construct collage using images and compass construction(50 minutes)

### Day One

- Demonstrate and construct pentagon and pentagram using compass and straightedge. (see page 6) (35 minutes)
- Complete phi worksheet (see page 7) (15 minutes)
- Reflect on activity
- Share reflections

### Day Two (Collage extension optional)

- Demonstrate and construct spirals with Golden Rectangles (Fibonacci numbers), using compass and straightedge. (30 minutes)
- Introduce collage and show examples (10 minutes)
- Select and cut images for collage (12 minutes)

### Day Three (Optional)

- Demonstrate gluing (3 minutes)
- Create collage –gluing images to substrate (20 minutes)

### Day Four (Optional: when collage is dry)

- Construct Golden Ratio image (pentagon/pentagram/spiral) on top of the collage (20 minutes)
- Reflection

## Ideas for Further Independent Student Project

- Historical research of the Golden Ratio
- Further geometric constructions
- Construction Golden Ratio images with Geometers Sketchpad

## Golden Ratio

*(measure each image to the nearest millimeter)*

Object	Width	Length	Ratio (larger/smaller)

**Do you see a reappearing value in your ratios?**

\_\_\_\_\_

**If so, what is it?**

\_\_\_\_\_

**Conclusion from discussion:**

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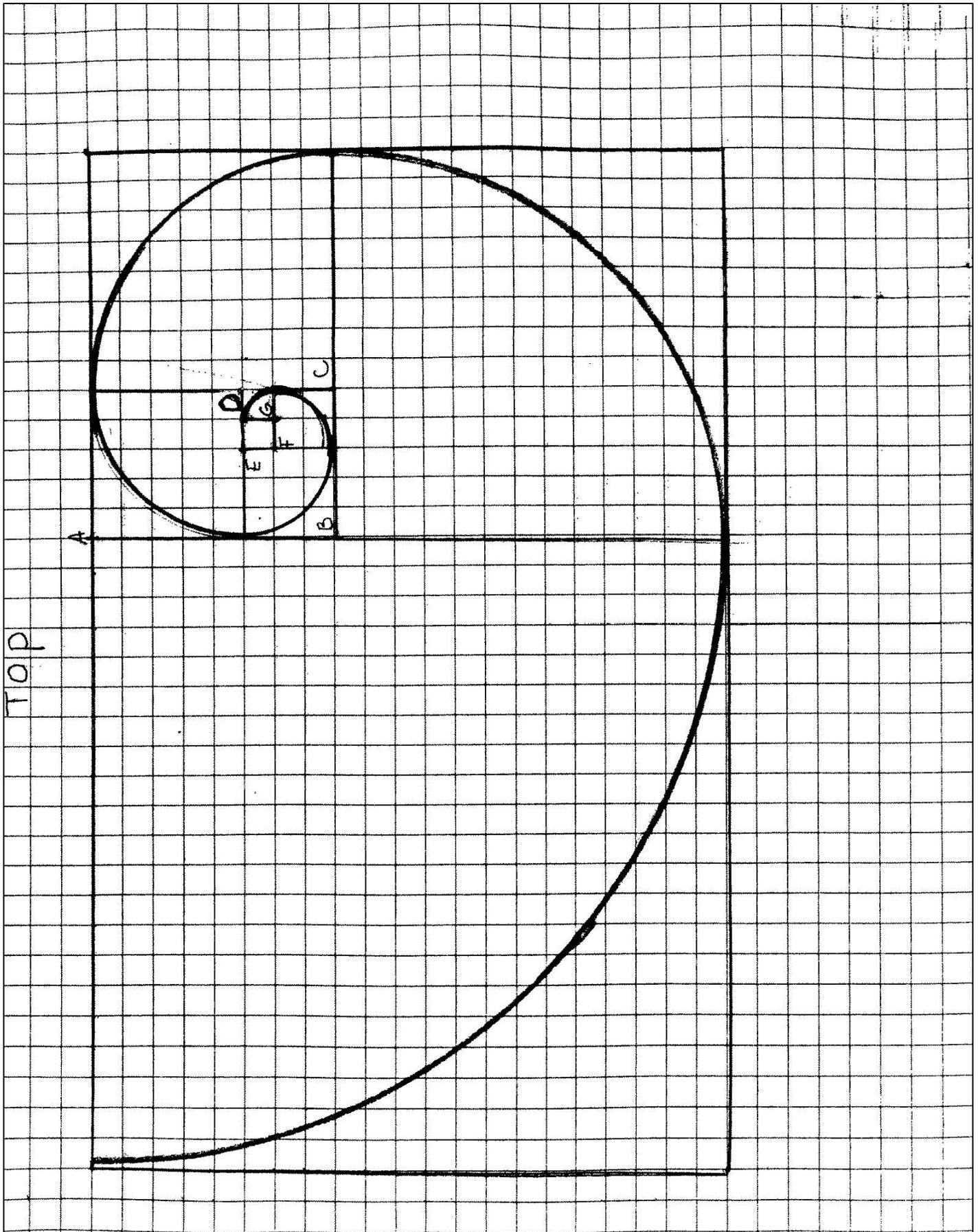
## Pentagon Construction

- Draw a segment
- Put the point of the compass on the segment and draw a circle so that it intersects the segment at two points—label the circle center  $O$  and the points on the diameter  $A$  and  $A'$ .
- Construct the perpendicular bisector of  $AA'$  and label one of the points of intersection on the circle  $S$
- Construct the perpendicular bisector of  $SO$  and find its intersection with the line  $SO$ ; call this  $M$ .
- Draw circle with center  $M$  with radius  $OM$ .
- Draw a line through  $A'$  and  $M$ . Find its intersections with circle  $M$ ; call these  $P$  and  $Q$
- Draw two circles with center  $A'$  and radii  $QA'$  and  $PA'$ . Find their intersections with circle  $O$ ; call these points  $B$ ,  $C$ ,  $D$ , and  $E$ .
- Draw segments  $AB$ ,  $BC$ ,  $CD$ ,  $DE$ , and  $EA$ .



## Golden Spiral: Part I

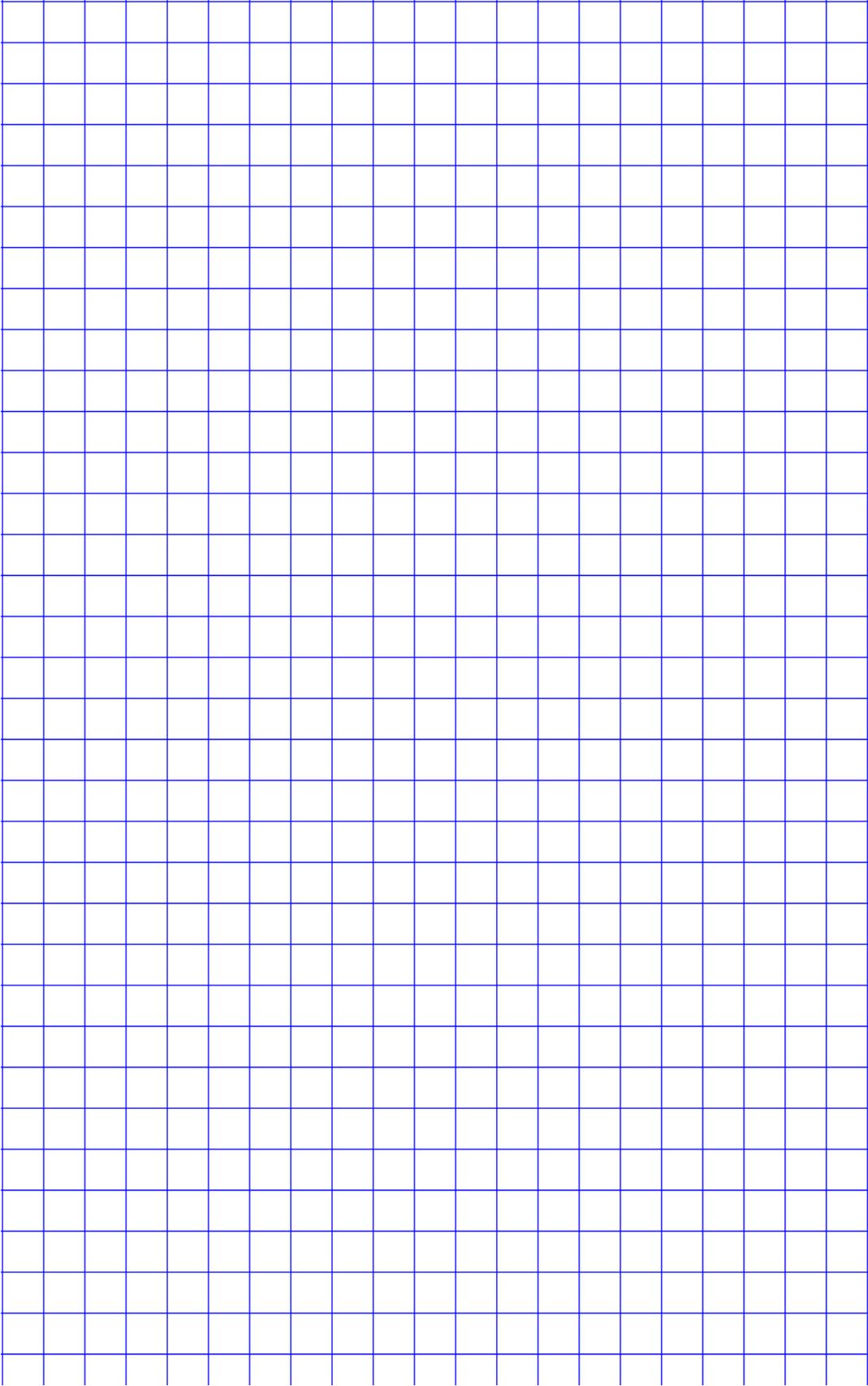
- On graph paper, mark a rectangle 34 X 21 units.
- In a landscape orientation, use the left side of the rectangle, for a side of the square, making a 21 X 21 unit square. Label the upper right corner A. This leaves a 13 X 21 rectangle on the right.
- In this rectangle, make a 13 X 13 square at the bottom. Label the upper left corner B.
- In the 8 X 13 rectangle, mark an 8 X 8 square on the right side. Label the lower left corner C.
- In the 5 X 8 rectangle, mark off a 5 X 5 square at the top. Label the lower right corner D.
- In the 5 X 3 rectangle, mark off a 3 X 3 square on the left. Label the upper right corner E.
- In the 2 X 3 rectangle, mark off a 2 X 2 square on the bottom. Label the upper left corner F.
- In the 2 X 1 rectangle, mark off a 1 X 1 square. On the right square, label the lower left corner G.



## **SPIRAL CONSTRUCTION: Part II**

- Open the compass to the length of the side of the 1 x 1 square and swing this arc from pt. G, creating this arc within the 1 x 1 square.
- Open the compass to the length of the side of the 2 x 2 square and swing this arc from pt. F, creating this arc within the 2 x 2 square.
- Open the compass to the length of the side of the 3 x 3 square and swing this arc from pt. E, creating this arc within the 3 x 3 square.
- Open the compass to the length of the side of the 5 x 5 square and swing this arc from pt. D, creating this arc within the 5 x 5 square.
- Open the compass to the length of the side of the 8 x 8 square and swing this arc from pt. C, creating this arc within the 8 x 8 square.
- Open the compass to the length of the side of the 13 x 13 square and swing this arc from pt. B, creating this arc within the 13 x 13 square.
- Open the compass to the length of the side of the 21 x 21 square and swing this arc from pt. A, creating this arc within the 21 x 21 square.

Geometry and Art: Golden Ratio



## Collage Techniques

Following these helpful hints will help you to arrange your elements and pieces so they will be both fun and aesthetically pleasing when you are finished. First, let's have a working definition of the word *Collage* and its foundations.

**Collage** - A [picture](#) or [design](#) created by [adhering](#) such basically [flat](#) elements as newspaper, [wallpaper](#), printed [text](#) and [illustrations](#), [photographs](#), cloth, string, etc., to a [flat surface](#), when the result becomes [three-dimensional](#), and *might* also be called a [relief sculpture](#) / [construction](#) / [assemblage](#). Most of the elements adhered in producing most collages are "[found](#)" materials. Introduced by the [Cubist](#) artists, this [process](#) was widely used by [artists](#) who followed, and is a familiar [technique](#) in [contemporary art](#).

"Collage" was originally a French word, derived from the word *coller*, meaning "to paste."

Now that you have a working knowledge of the word *collage*, it is time to put your collage together. Arrangement is at the heart of the principles of design, and its consideration determines a work's [coherence](#) ([unity](#) and [variety](#)), [focal point](#), [rhythm](#), etc. These elements work well if you remember a few important design tips while constructing your work of art.

Here are a few suggestions for the arrangement of your collage pieces to help you to be more successful.

1. While choosing the pictures and items you would like to use, remember to make them unique. You can cut some out following their contours, or you can actually tear pieces.
2. Things that are arranged in odd numbers on the surface will always be more interesting than if you use even numbers. Therefore, work with groups of three items to create your initial arrangement.
3. Remember to arrange your pieces so that the viewer's eye must travel throughout the entire piece. From time to time, stand back and look at the piece. Allow your vision to follow areas of interest, and note any areas that your eye does not travel to. Rearrange until you are satisfied that your viewer will experience the entire piece when it is finally viewed by your audience.
4. Once you have decided where you want your pieces and elements to go, number them both on the pieces and on the background paper. This will help to keep the continuity while you are removing pieces to rubber cement them down.
5. The next step will be to use the neutral colored tissue paper as a glaze or muting technique to help to bring all the elements together. Rip up pieces of the tissue into different shapes to lay on your piece. Use more than one color of tissue and again, arrange them on your piece the same way you worked with your original pieces.

## Geometry and Art: Golden Ratio

6. Next, using a mixture of white glue and water, (about half/half), glue them in place. You can use the leftover glue glaze to brush over the entire piece, which will leave you with a gentle opaque over glaze.
7. Sign your work, and you are done!

Don't be overcritical of your work and the process. Many times, what you think will be a flaw will end up being one of the best points of interest. So don't be too concerned with controlling each and every aspect of your work. Enjoy the process, and celebrate your successes with your students!

### Note: Golden Ratio

Phi represents the ratio of two successive Fibonacci numbers. It is evident around us as the ratio of total length to portion of the length. An example of this is the total height of the space needle to the height of the observation deck. Reviewing measuring skills, in millimeters, is important.

If you are only emphasizing compass constructions, you may just want to cover Day One. However, the collage project can provide some amazing results!

## A Math Teacher's Work



## Assessment

### Rubric

Name \_\_\_\_\_

Period: \_\_\_\_\_

## Collage Grade Sheet

### Objectives:

1. Cut a variety of Golden Ratio images to glue together to form a collage.
2. Glue the images onto a background making sure that it is pleasing and neatly done.
3. Optional: Glue tissue paper over the collage to create emphasis and added depth.
4. Construct geometric forms showing the Golden Ratio (spiral or pentagram). Over the collage.
5. Use contrast of values to emphasize the Golden Ratio.

1. How did you illustrate the Golden Ratio in your collage? \_\_\_\_\_

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2. Where were you most surprised to see the Golden Ratio? \_\_\_\_\_

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3. What connections do you see between geometry and art? \_\_\_\_\_

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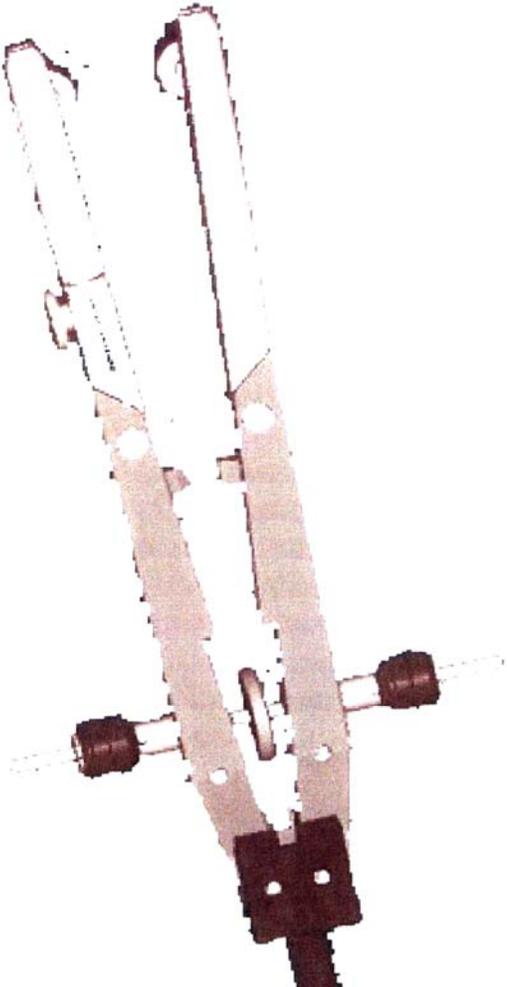
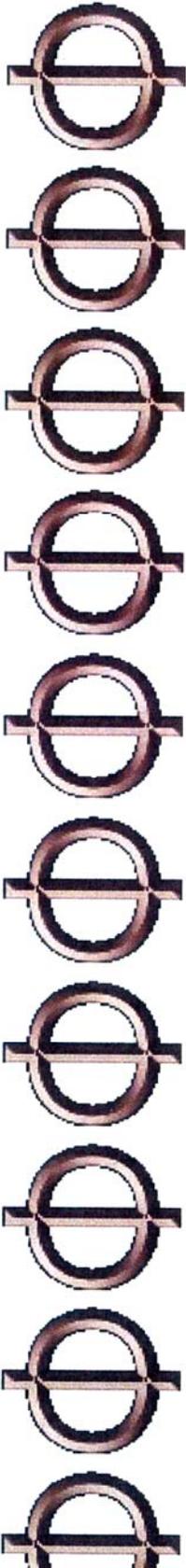
4. Explain something you learned about geometry and art doing this project. \_\_\_\_\_

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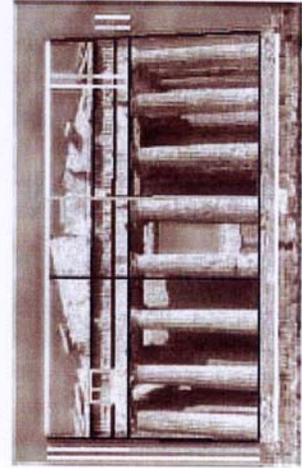
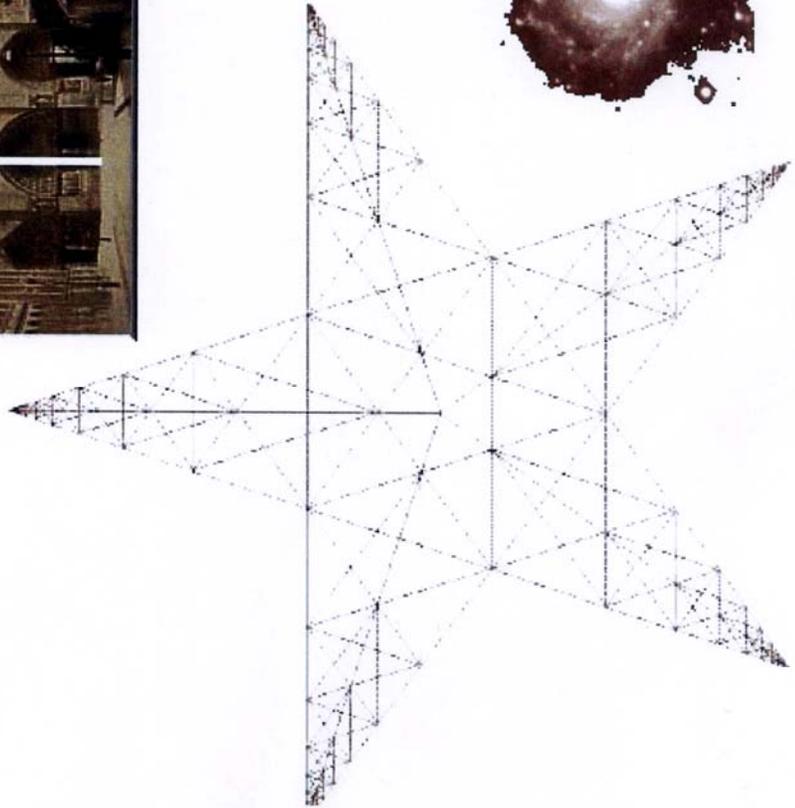
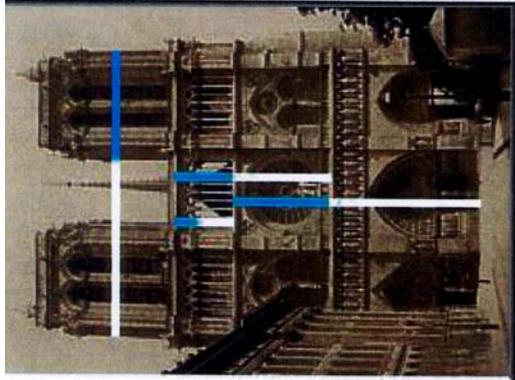
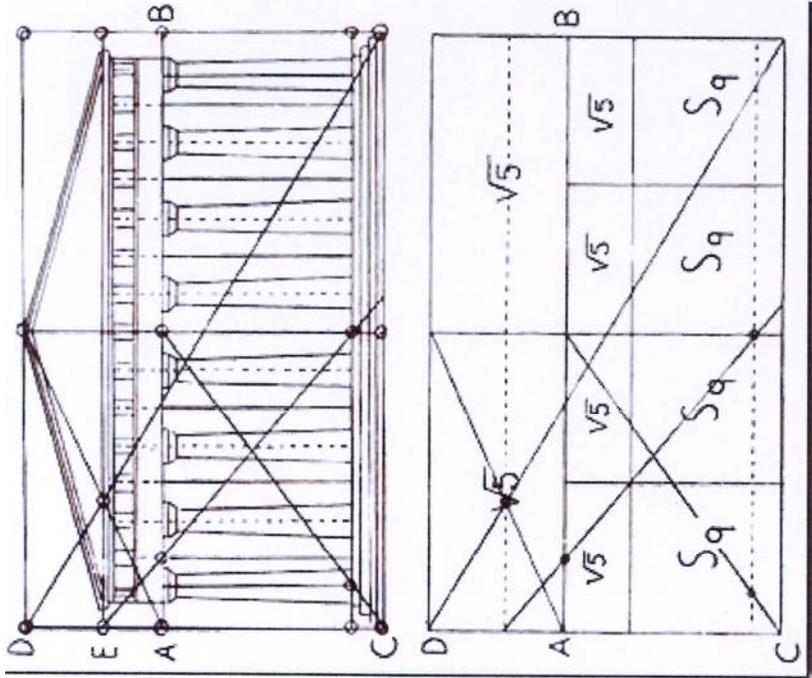
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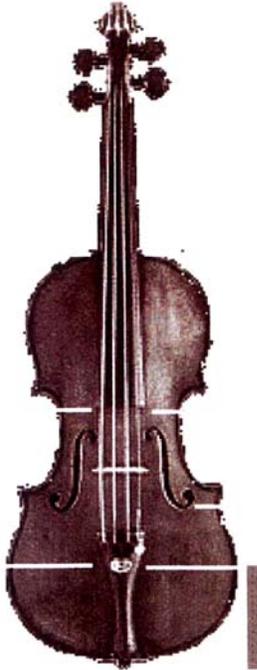
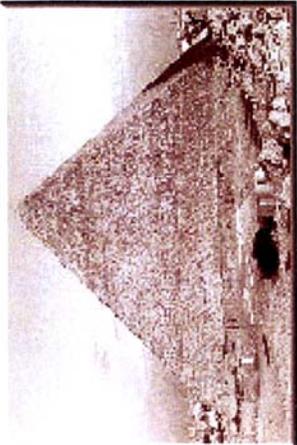
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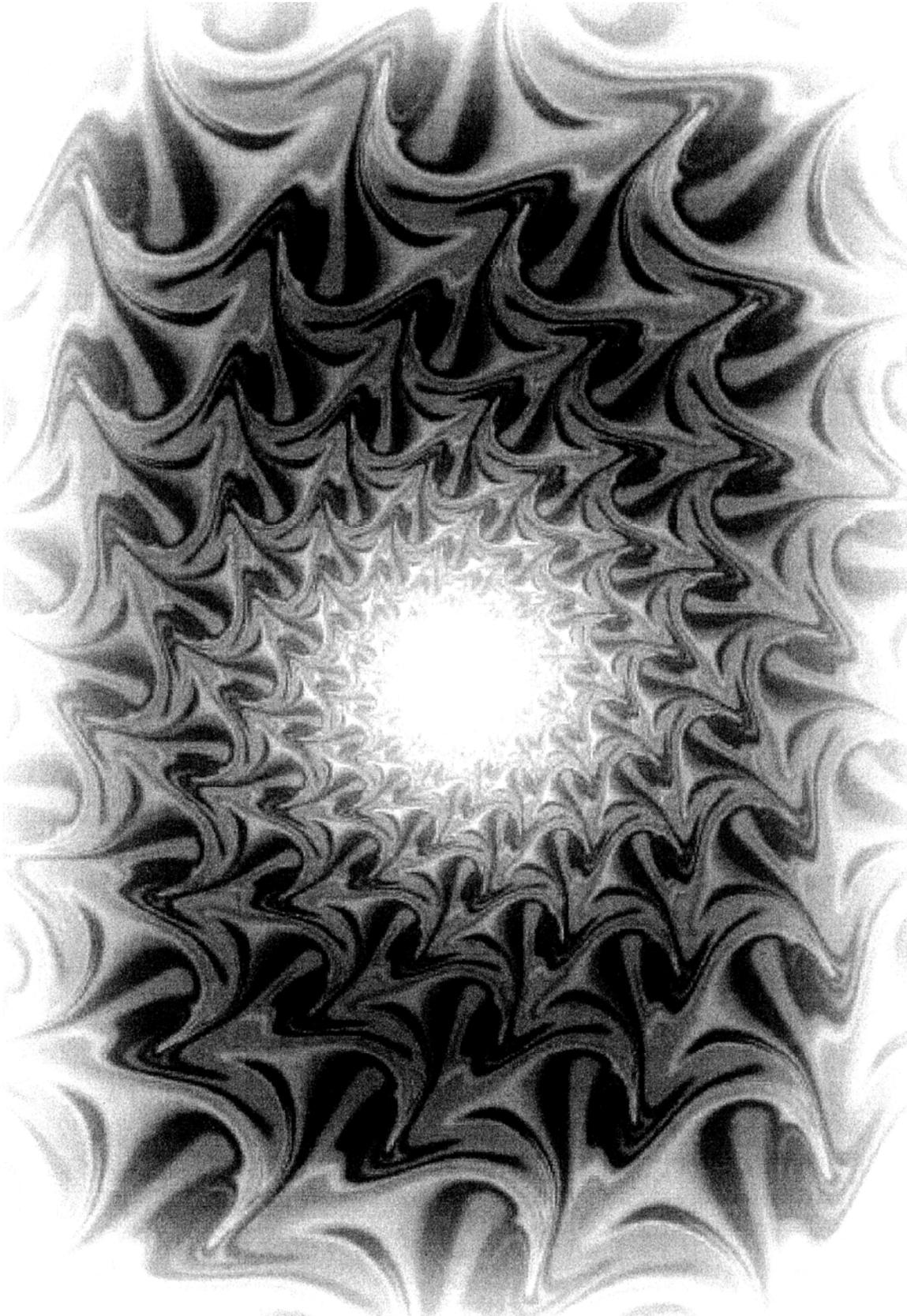




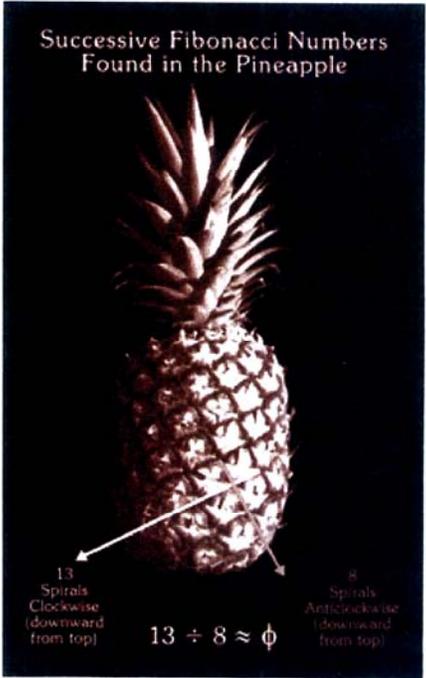
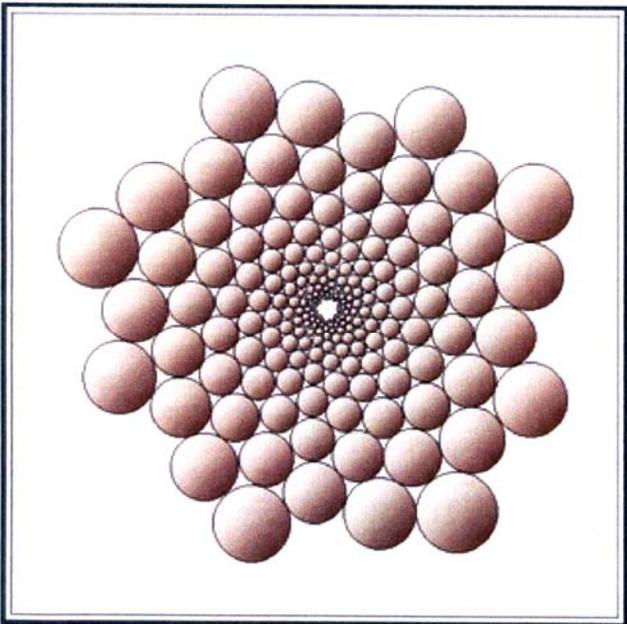
Geometry and Art: Golden Ratio

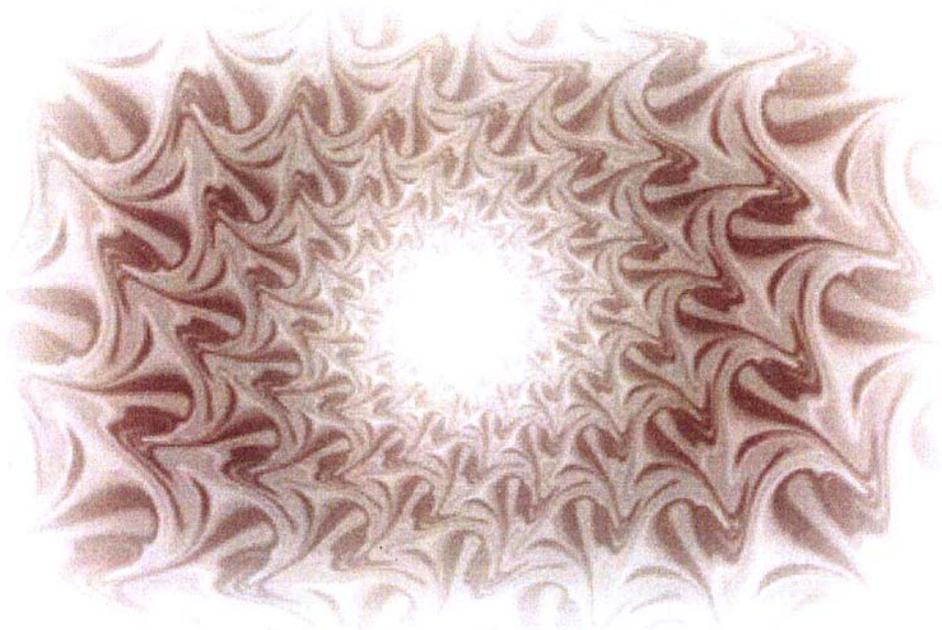


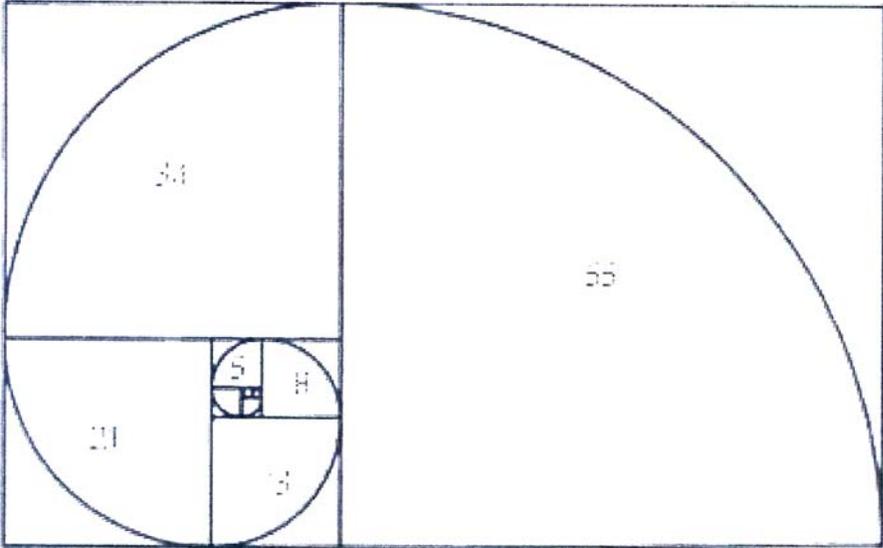
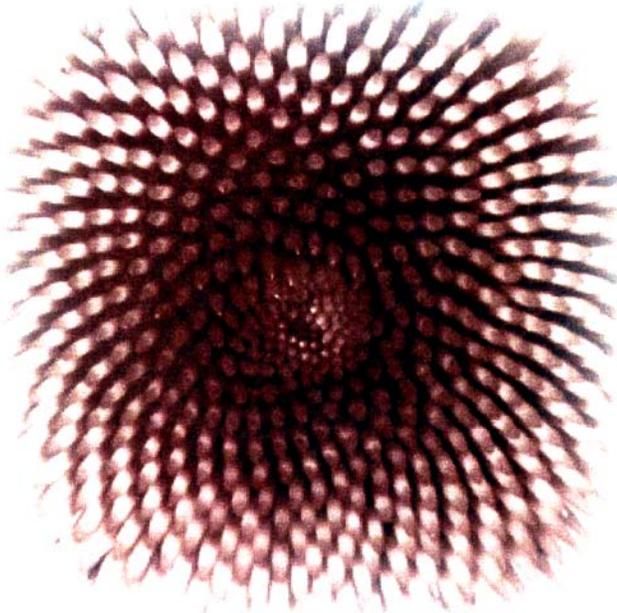
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Geometry and Art: Golden Ratio







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