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Using a Learning Progression to Formatively Assess the Concept of Slope

Smarter Balanced Digital Library Webinar
April 16, 2015

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In this Webinar

- Project background
- A learning progression for linear functions and its use in formative assessment
- Smarter Balanced assessment targets addressed by these formative tasks
- Excerpts from a formative task
- Teacher Handbook for formative tasks

Goals for this Webinar

- To illustrate how learning progressions could be used to help identify at what level a student understands slope
- To illustrate how the tasks in the digital library can be used to assist students in advancing their understanding of constant change as described in the learning progression

The CBAL™ Initiative

Cognitively-Based Assessment *of*, *for*, and *as* Learning

- "*of*, *for*, and *as* Learning" — intended to bring learning sciences principles into assessment and to integrate learning and assessment as a process that results in a worthwhile experience for students and teachers
- Tasks provide a tool for teachers to use to model, encourage, and assess *mathematical thinking*
- Used in conjunction with local curricula
- Focus on select big ideas in mathematics
- Utilize learning progressions

Definition of a Learning Progression (in CBAL)

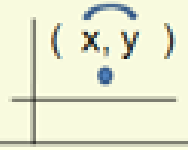
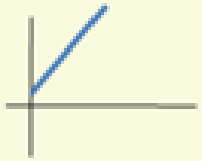
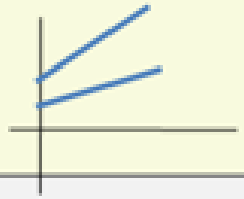
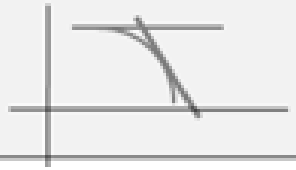
- Description of qualitative change in student's level of sophistication for a key concept, process, strategy, practice, or habit of mind
- Change in student standing on progression may be due to variety of factors, including maturation and instruction
- Each progression is assumed to hold for most, but not all, students

Learning Progressions - Potential Value Added

- Introduce new insights into how students may understand major mathematical concepts
- Connect student thinking with evidence (work) produced
- Suggest possible next steps for instruction, based on descriptors of levels in learning progression

Linear Functions Learning Progression

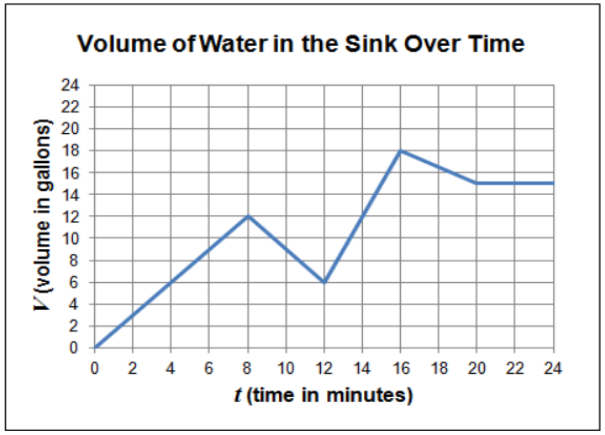
(More details in Teacher Handbook)

Level 1	Separate representations [numeric, spatial, symbolic]	One dimensional change	
Level 2	**Coordinate plane**	**Mutual change**	
Level 3	**Linearity** $y = mx$ and $y = mx+b$	**Constant change**	
Level 4	**More than one linear function**	**Comparing changes**	
Level 5	Non-linear functions	Changing change	

Connecting LP to questions

CBAL

Question 4 of 4



Water Flows In Faucet is turned on:			Water Flows Out
Full	Halfway	Quarter of the way	Plug out
3 gallons per minute	1.5 gallons per minute	0.75 gallons per minute	1.5 gallons per minute

Describe the sink conditions from 0 to 24 minutes as indicated by each segment of the graph.

From 0 minutes to 8 minutes, the faucet is Select...
the plug is Select...

From 8 minutes to 12 minutes, the faucet is Select...
the plug is Select...

From 12 minutes to 16 minutes, the faucet is Select...
the plug is Select...

From 16 minutes to 20 minutes, the faucet is Select...
the plug is Select...

From 20 minutes to 24 minutes, the faucet is Select...
the plug is Select...

Level 4	More than one linear function	Comparing changes	

Connecting Tasks to Smarter Balanced Assessment Targets

Tasks in this unit assess portions of the following targets.

- Grade 7 Target D – Solve real-life and mathematical problems using numerical and algebraic expressions and equations.
- Grade 8 Target D – Define, evaluate, and compare functions.
- Grade 8 Target F – Use functions to model relationships between quantities.
- Grade 8 Target J – Investigate patterns of association in bivariate data.
- High School Target G – Create equations that describe numbers or relationships.
- High School Target I – Solve equations and inequalities in one variable.
- High School Target L – Interpret functions that arise in applications in terms of the context.

CBAL Materials in the Digital Library

- Two units of materials – one on proportional reasoning; other on slope and linear functions
- Each unit contains
  - Two digital tasks – teacher accesses them online; creates opportunities for teacher/student discourse
  - Teacher Handbooks for each task – PDF files
  - Videos (for teachers) for the proportional reasoning unit – further explain the proportional reasoning learning progression
- Featured in this session – one task from the slope and linear functions unit

How to Use the Materials

- Tasks are designed to encourage and support student/teacher interaction.
- Divided into sections – can incorporate over a period of several days or weeks with local curriculum and materials
- Project a section of a task on a Smart board or screen, and engage the class in discussion using guidance provided in Teacher Handbook.

Excerpts from *Dams and Droughts* Formative Assessment Task

Linear Functions Learning Progression

(More details on it in Teacher Handbook)

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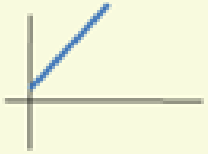
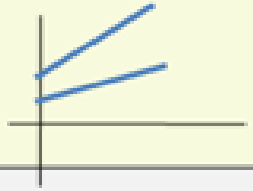
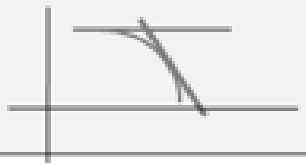
Level 3	**Linearity** $y = mx$ **and** $y = mx+b$	**Constant change**	
Level 4	**More than one linear function**	**Comparing changes**	
Level 5	Non-linear functions	Changing change	

Dams and Droughts

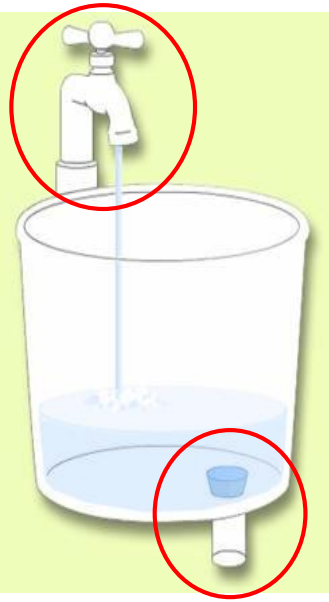
This picture shows a lake that had a water crisis in 2007.

The lake has a dam at one end. Water flowing from the lake past the dam is used to create electricity and provide water for crops. The lake may become so shallow that there will not be enough water to generate electricity.

In the first part of the task, you will study how a sink works in order to understand how a dam works. In the second part of the task, you will analyze data on water crises to determine whether action should be taken to reduce loss of water in the lake.

Will the water crisis that started in January, 2009, last long enough for the water level to become too low to generate electricity?

How a Dam and the Lake Behind it Work



Like the plug in the sink, the dam forms the barrier that prevents water from flowing out of the lake.

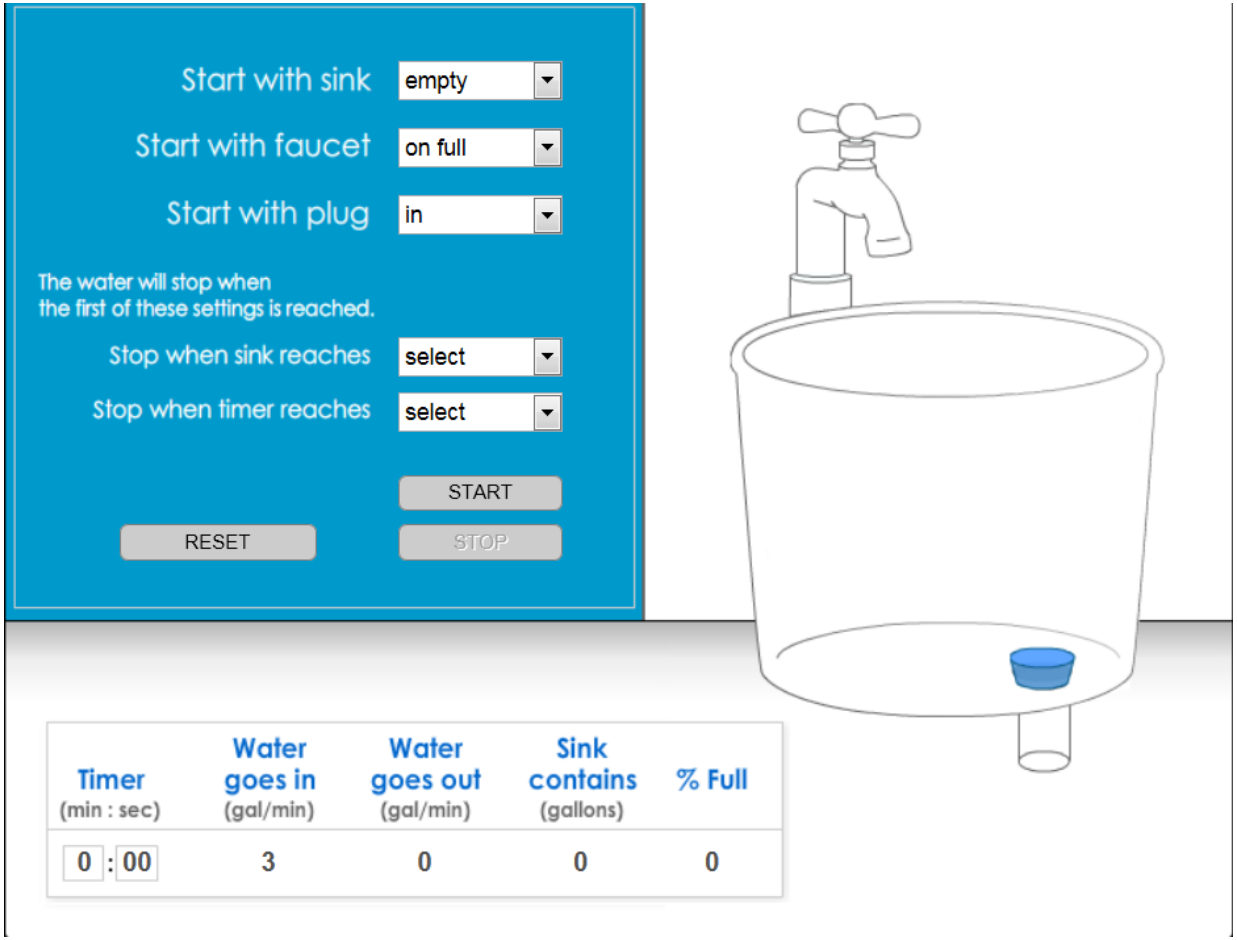
As in the faucet, water flows from the river to form a lake behind the dam.

Practice
In the table, match the parts from the sink to the corresponding parts in the photo.

Faucet	select
Sink	select
Plug	select

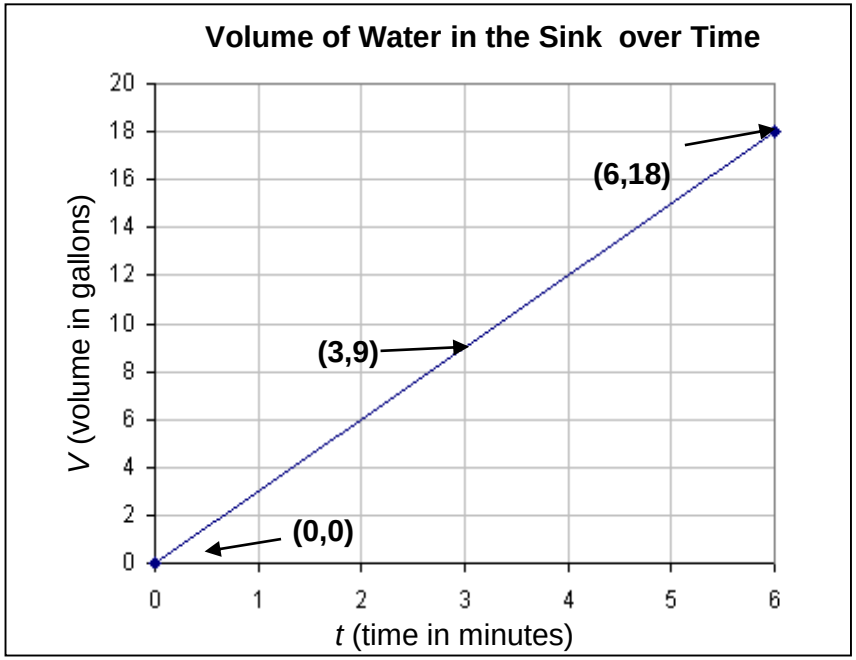
Click here for correct answers

The Sink Tool



The line on the graph shows the relationship between the volume (gallons), *V,* of water in the sink and time (minutes), *t*, for certain sink conditions. These sink conditions correspond to the data in the table below.

Water Flows In			Water Flows Out
Faucet is turned on:			
All the way	Halfway	Quarter of the way	Plug out
3 gallons per minute	1.5 gallons per minute	0.75 gallons per minute	1.5 gallons per minute



The slope of the line is 3 gallons/minute.

a. What does the slope tell you about what is happening to the volume of water in the sink over time?

b. Since the slope is 3 gallons/minute,

the faucet is

and the plug is

c. The sink starts

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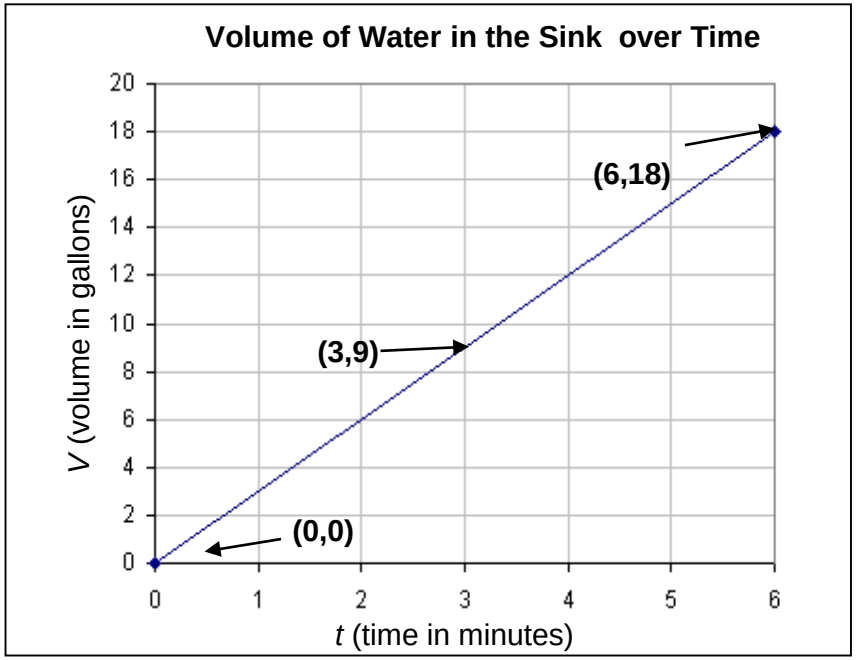

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b. Since the slope is 3 gallons/minute,

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c. The sink starts

Possible evidence of Level 2 on Learning Progression
a. "At 3 minutes there are 9 gallons."

Possible evidence of Level 3 on Learning Progression
a. "It tells me that the sink increases as time passes. So, there are three gal of water going in every minute."

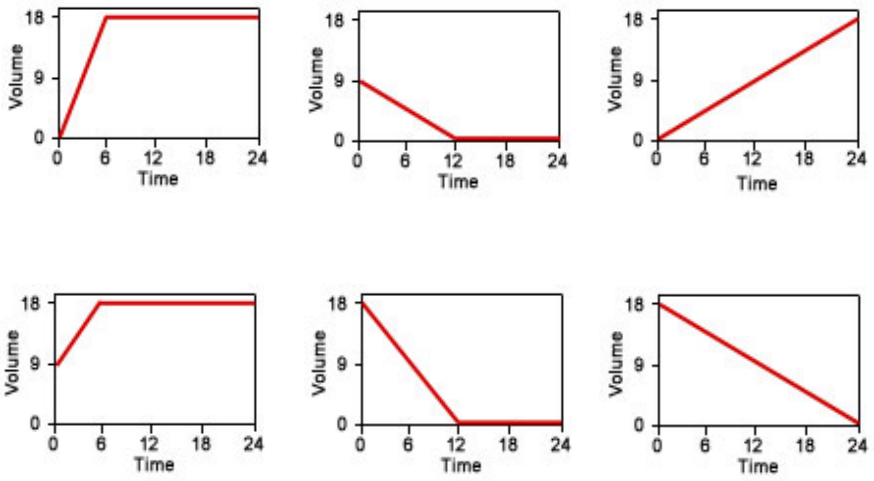
Complete each row in the table by clicking the graph that represents the conditions in that row, then clicking in the corresponding cell.

	Sink starts:	Faucet is turned:	Plug is:	Graph
a.	empty	on one-quarter of the way	in	
b.	half full	on halfway	in	
c.	full	off	out	
d.	full	on one-quarter of the way	out	

A full sink holds 18 gallons of water. The rate at which the water flows in or out is shown below.

Water Flows In			Water Flows Out
Faucet is turned on all the way	Faucet is turned on halfway	Faucet is turned on one-quarter of the way	Plug out
3 gallons per minute	1.5 gallons per minute	0.75 gallons per minute	1.5 gallons per minute

Volume (gallons) of Water in the Sink in terms of Time (minutes)



Part 2: The Lake

The photos of the lake shown were taken 18 months apart.

© fotoVoyager/istockPhoto#2563244

© MICHELANGELOBOY/istockPhoto#10652896

A comparison of the photos shows that the water level has dropped and that the amount of water in the lake has decreased. If the water level continues to drop, there may eventually be too little water in the lake to generate electricity. In this section of the task, you will analyze data on water crises to determine whether water conservation actions should to be taken to help reduce loss of water in the lake.

Will the water crisis that started in January, 2009, last long enough for the water level to become too low to generate electricity?

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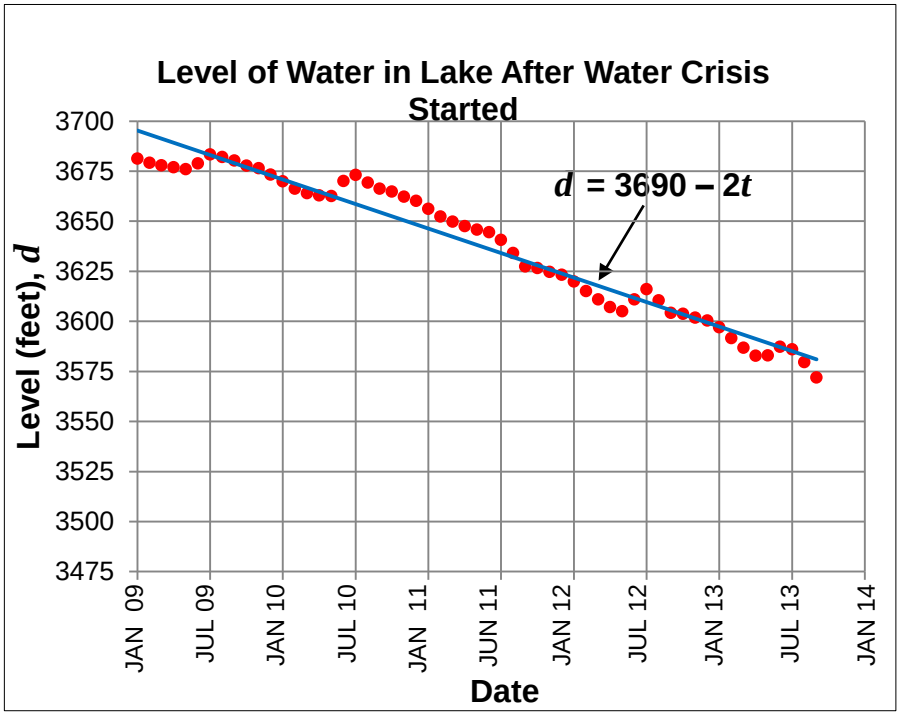


The data can be modeled by the blue line and its equation below.

$d = 3690 - 2t$

where d = level of water in the lake (feet)
t = number of months since Jan. 2009

a. What is the *y*-intercept in the equation?

b. What does it represent in this situation?

Drop down menu has these choices:

The level of water in the lake in January 2009

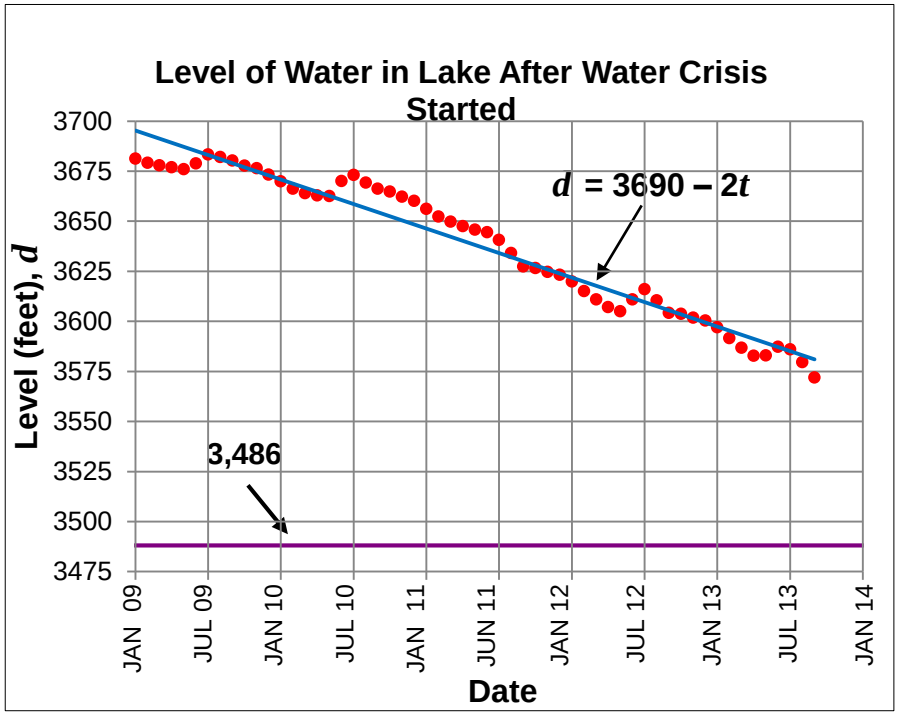
How much water is in the lake

Time the graph intersects with the y axis

The lake begins with 3690 feet of water.

Every month you go down 2 feet.

Will the water crisis that started in January, 2009, last long enough for the water level to become too low to generate electricity?



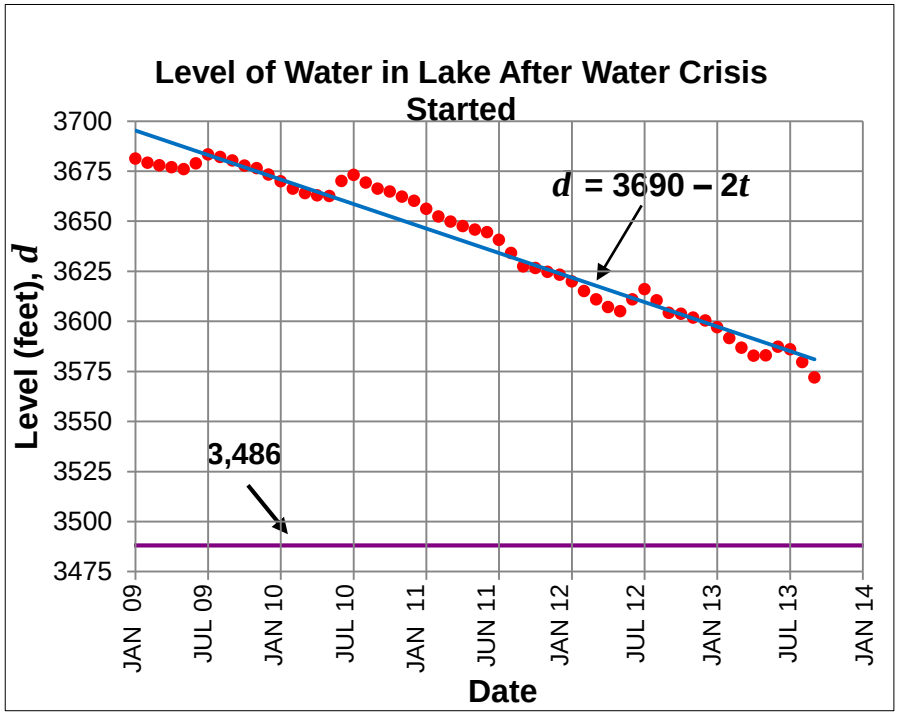
Use the equation $d = 3690 - 2t$ to determine how long it will take the level of water in the lake to reach 3,486 feet, the level at which electricity will no longer be generated.

Show your work.

$t =$ months from beginning of Jan. **2009**

Click here to calculate the date

Will the water crisis that started in January, 2009, last long enough for the water level to become too low to generate electricity?



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Show your work.

$t =$ months from beginning of Jan. **2009**

Click here to calculate the date

Possible evidence of Level 3, moving to Level 4 (since student is dealing with different representations)

"3486 = 3690 – 2t -204 = -2t t = 102

102 months from Jan 09."

Culminating Section
Students analyze drought data and summary statistics from past 100 years.

In **December 2013**, a politician stated:
"There is no need for water conservation actions since there is no major water crisis. There is no chance that this water crisis will last long enough so that not enough water will be available to generate electricity."

A group of concerned citizens disagree and present the model that shows the crisis lasting until July 2017. They also present data they collected about other water crises in the lake along with the summary statistics.

Data collected about water crises for the area

Years in water crisis	1907 - 1916	1933 - 1937	1953 - 1961	1964 - 1967	1990 - 1996
Number of years in water crisis	10	5	9	4	7

Summary statistics

	Mean	Max	Min	Range
Number of years in water crisis	7	10	4	6

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Number of years in water crisis	10	5	9	4	7

Summary statistics

	Mean	Max	Min	Range
Number of years in water crisis	7	10	4	6

Explain how the group can use the mean, maximum, minimum and/ or range to convince the politician that there is a reasonable chance that the current crisis could last 8 ½ years, long enough for the level of water in the lake to drop so far that the dam will no longer produce electricity.

Students formulate a data based justification regarding the possibility of a water crisis occurring in the next few years.

Support for Teachers – CBAL Teacher Handbooks

- Intended to support good learning and teaching
- Organized to help focus on what you need to know as you are using the task

Discussion for each question includes

- Correct response,
- What is being assessed,
- What to emphasize to students, and
- What to look for in student responses (i.e., interpreting the evidence)

Teacher Handbooks

Connects formative assessment to Assessment for Learning

Connecting CBAL Formative Assessment to the Five Guiding Principles of Assessment for Learning

Definition of Formative Assessment for the CBAL Project

Formative assessment is an ongoing process in which teachers and students use evidence gathered through formal and informal means to make inferences about student competency and, based on those inferences, take actions intended to achieve learning goals. Five guiding principles of assessment for learning play an important role in formative assessment on CBAL. Those principles appear below.

Five Guiding Principles of Assessment for Learning

1. ***Sharing Learning Expectations:*** Clarifying and sharing learning intentions and criteria for success
2. ***Questioning:*** Engineering effective classroom discussions, questions, and learning tasks that elicit evidence of learning
3. ***Feedback:*** Providing feedback that moves learners forward
4. ***Self-Assessment:*** Activating students as the owners of their own learning
5. ***Peer Assessment:*** Activating students as instructional resources for one another Clarifying and Sharing Learning Intentions and Criteria for Success

Three related questions about student learning help connect each of the Five Guiding Principles to the definition of formative assessment for the CBAL project. They are: "Where is the learner going?" "Where is the learner right now?" and "How does the learner close the gap?"

Each of the five principles is explained in greater detail below.

Teacher Handbooks

Dams and Droughts

Background for this Task

This first part of this task focuses on linear relationships among rate, time, and volume. A sink simulation allows students to control the rate at which a sink fills or empties so they can explore slope, *y*-intercept, and endpoints of a graph in an authentic setting. Students will work with graphs and equations that represent volume as a function time.

The second part provides experience for the student in using the cross-cutting processes of model, argue, and represent within the competency of understanding and using algebra. There are also some questions from the understanding and using data analysis and probability competency. Students are presented with a setting for the task that involves an area of the country facing a possible water crisis. Data and information relative to the water level in a large lake are available in tables and in graphs for the student to use in supporting an argument. The task steps the student through the data in each table and graph before asking him or her to put all of the information together at the end of the task in support of a claim. A guide tool is available to help students read graphs of water levels over time and a dam tool is available to students to help them see how a dam works and what happens if the water level in the lake becomes too low to generate electricity.

The student should take time to become familiar with the tools in this task since they provide useful and accessible models for better understanding how the water in the lake and the dam relate to each other.

A high level overview provides a general orientation to the task, explaining its goals and suggesting hints for successful implementation.

Part One, Section 1/*Plug In*: Question 5

***Correct response*:**
(a) Sample response: *It is increasing by 3 gallons every minute.*
(b) The faucet is on all the way and the plug is in.
(c) The sink starts empty.

***What is being assessed*:**
This question assesses whether the student can make the connection between the meaning of slope in the context of the task, the graph of the line, and the conditions in the sink (i.e., how much water is flowing from the faucet, whether the plug is in or out, and whether the sink starts empty or full). It provides evidence of level 3 of the *Functions/Linear Functions Learning Progression.*

***What to emphasize to students*:**
When graphs are presented in context, the student should realize that an important part of interpreting the graph correctly means understanding not only the mathematical information in the graph (in this instance, the linear relationship between volume of water in gallons and time in minutes), but also how that information relates to the situation or context that is represented by the graph. Labels on the axes of graphs, as well as graph titles, all work together to give important information about the data that are displayed and students need to realize that they should understand all parts of the graph when interpreting the information it displays.

***What to look for in student responses*:**
The slope of the line represents the net result of water flowing into the sink minus water flowing out of the sink. If the student correctly provided an answer of 3 gallons per minute in the preceding question, then in this question he or she should be looking for the combination of faucet setting and plug setting that will yield 3 gallons per minute. Since the possible faucet settings (in gallons per minute) are 3, 1.5, and 0.75, and the possible plug settings are 0 and 1.5 gallons per minute, the only combination that will yield 3 is 3 for the faucet (i.e., faucet on all the way) and 0 for the plug (i.e., plug in). Students should link the positive slope of the line to the fact that the volume of water in the sink is increasing, and further, because the slope is 3, the volume is increasing at a rate of 3 gallons per minute. They need to make both connections (i.e., between the positive value of the slope, as well as its numerical value) to fully understand the meaning of the slope in this situation.

Each question includes annotations that support good teaching and learning practices.

Teacher Handbooks

Following Up

In this set of tasks, students worked with different representations of functions. Those representations included graphs, tables, algebraic equations, and descriptions in words. Students should understand that functions are a tool for modeling real-world phenomena mathematically, and can enhance the understanding and study of those phenomena. Although the majority of the early study of functions is limited to linear types (thus, the emphasis on understanding thoroughly the slope and vertical intercept of a line in the context being modeled by the function), students should realize that nonlinear functions also exist, and that they have the understanding and knowledge to analyze some behavior of nonlinear functions (e.g., increasing/decreasing, maximum and minimum points, etc.).

As students continue their study of mathematics, their instruction should continue to afford them appropriate opportunities to

- Make connections among various mathematical representations (graphs, equations, tables, words),
- Construct interpretations of mathematical models in the context of the problem that is being studied or analyzed,
- Pay attention to details of graphs such as titles, axes labels and scales, maximum and minimum values, and trends displayed by the data in analyzing the behavior of functions and graphs, and
- Work with multiple mathematical stimuli (graphs, tables, equations) to generate complete analyses and complete mathematical arguments that support conclusions or claims.

Developing necessary facility with these practices will help to strengthen their levels of proficiency with and understanding of key mathematical concepts and ideas.

Discussion of tasks in each unit concludes with final reflections on the most important aspects of learning, as well as assessment and instructional implications moving forward.

Invitation to Follow-up

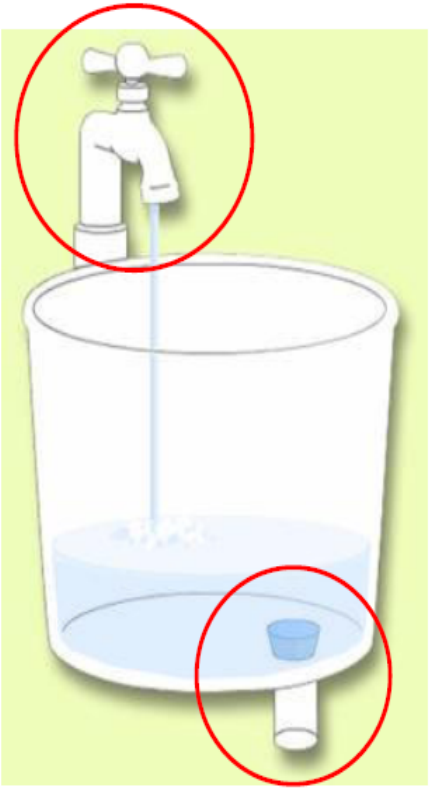
- After this webinar, complete the questions in Part One/Section 3 of Dams and Droughts
  - If you find your students are experiencing minimal difficulty with the above part/section, you may want to try Part Two/Sections 1 and, 2, and 3, OR
  - If you find your students are experiencing more difficulty than expected with the above part/section, you may want to try Sections 1 and 2 of Part One to help identify where the problems are.
- Share your experience in a forum during the week of May 4

Getting Started

1. Complete the task yourself.
2. Read through the corresponding sections of the Teacher Handbook.
3. Determine which implementation would work best in your class (e.g., Projected image from your laptop; whole group instruction and/or small group instruction, etc.).
4. If you elect to use only Section 3 of Part One of the task with your students, be sure to introduce them to that section using the PDF of the next slide. That slide is meant to familiarize students with the sinks tool and is provided within the DL unit of materials.
5. When using the task in class, be sure to engage your students in the discussion. See Teacher Handbook for guidance on questioning techniques.
6. Begin to think about at which level on the LP your students may be located.
7. What will you do to help students advance in their understanding to the next LP level? (What would be a logical next instructional step?)

Rate of Water Flow Into and Out of a Sink

Water flows from the faucet into the sink.



The sink holds 18 gallons of water when it is full.

Water can flow into the sink at one of three different rates, depending on how much the faucet is turned on. See the table below.

When the plug is out of the drain hole, water flows out of the sink at 1.5 gallons per minute.

Water Flows In			Water Flows Out
Faucet is turned on all the way	Faucet is turned on halfway	Faucet is turned on one-quarter of the way	Plug out
3 gallons per minute	1.5 gallons per minute	0.75 gallons per minute	1.5 gallons per minute

The plug is in the drain hole. It prevents water from flowing out of the sink.

Spotlight Forum
May 4-8

- Navigate to *Linear Functions And Nonlinearity: CBAL Activity Set And Handbook* in the Digital Library.
- Click on the Collaboration tab below the preview window.
- Click on the *Spotlight Forum May 4-8* card.
- Join the conversation.