

MAT.HS.PT.4.HMOFC.A.268

Sample Item ID:	MAT.HS.PT.4.HMOFC.A.268
Title:	Home Office
Grade:	HS
Primary Claim:	Claim 4: Modeling and Data Analysis Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.
Secondary Claim(s):	Claim 1: Concepts and Procedures Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency. Claim 2: Problem Solving Students can solve a range of well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies.
Primary Content Domain	Number and Quantity
Secondary Content Domain(s):	Geometry, Functions, Algebra
Assessment Target(s):	4 A: Apply mathematics to solve problems arising in everyday life, society, and the workplace. 4 B: Construct, autonomously, chains of reasoning to justify mathematical models used, interpretations made, and solutions proposed for a complex problem. 4 G: Identify, analyze, and synthesize relevant external resources to pose or solve problems. 4 D: Interpret results in the context of a situation. 1 C: Reason quantitatively and use units to solve problems. 1 G: Create equations that describe numbers or relationships. 1 E (Gr 7): Draw, construct, and describe geometrical figures and describe the relationships between them. 1 F (Gr 7): Solve real-life and mathematical problems involving angle measure, area, surface area, and volume. 1 A (Gr 7): Analyze proportional relationships and use them to solve real-world and mathematical problems. 1 D (Gr 7): Solve real-life and mathematical problems using numerical and algebraic expressions and equations. 1 J (Gr 6): Summarize and describe distributions.

	2 B: Select and use appropriate tools strategically.
Standard(s):	N-Q.1, N-Q.3, A-CED.1, A-REI.4, F-IF.2, F-BF.2, F-LE.2, G-SRT.8, 7.G.1, 7.G.6, 7.RP.3, 7.EE.3, 6.SP.5
Mathematical Practice(s):	1, 2, 3, 4, 5, 6, 7
DOK:	3
Item Type:	PT
Score Points:	16
Difficulty:	M
How this task addresses the “sufficient evidence” for this claim:	The student uses concepts of geometry, functions, algebraic thinking, and number sense to accomplish tasks associated with having a home office built and identifying amounts that can be used as deductions based on the area of the home office. The work is supported by calculations and explanations of reasoning.
Target-specific attributes (e.g., accessibility issues):	Accommodations may be necessary for students who have vision challenges, fine motor-skill challenges, and language-processing challenges.
Stimulus/Source:	Sources used for flooring prices in Flooring Options table: http://www.homedepot.com/webapp/catalog/servlet/Search?keyword=carpet+prices&selectedCatgry=SEARCH+ALL&langId=-1&storeId=10051&catalogId=10053&Ns=None&Ntpr=1&Ntpc=1#/?c=1&rpp=96 http://www.homedepot.com/webapp/catalog/servlet/Search?keyword=tile+prices&selectedCatgry=SEARCH+ALL&langId=-1&storeId=10051&catalogId=10053&Ns=None&Ntpr=1&Ntpc=1 http://www.homedepot.com/webapp/catalog/servlet/Search?keyword=laminate+prices&selectedCatgry=SEARCH+ALL&langId=-1&storeId=10051&catalogId=10053&Ns=None&Ntpr=1&Ntpc=1 Source used for tax details: http://www.irs.gov/newsroom/article/0,,id=108138,00.html
Notes:	Multi-Part Task
Task Overview:	Students will calculate the area of a home office space in a finished basement, given a set budget amount. Then students will decide on a type of flooring to use for the home office given a set of flooring options and then calculate its cost. They will relate the area of the home office to utility expenses for the entire house to predict a tax deduction amount for the use of the home office. All parts, A through D, will be scored for this task.
Teacher preparation/Resource requirements:	Teacher preparation: At least a day or two prior to starting this task, the teacher should put together a “Flooring Options” table which shows some different types of flooring and at least three sample costs and related sizes or measured units for each of these options. The table below can be used until these costs are no longer viable (outdated). This table is the same as the one used in the

	<p>Sample Top-Score Response. This table will be given to students to use in <i>Part B</i> of the task. The table should include costs given in different units (e.g., cost per sq ft vs. sq yd) to have the student make use of conversion skills. Point this out to students so they are aware that conversions will be necessary.</p> <p>Resource requirements: Spreadsheet software and graphing paper for calculation work and diagram manipulation/work must be available to all students. Calculators should be available to students, either online or physically.</p>
Teacher Responsibilities During Administration:	Monitor individual student work and facilitate class discussion at the beginning of Session 1. Provide resources as necessary.
Time Requirements:	Two sessions totaling no more than 120 minutes; <i>Parts A</i> and <i>B</i> will be completed during Session 1. <i>Parts C</i> and <i>D</i> will be completed during Session 2. All parts during both sessions will be performed individually.

Prior to actually starting this task, the teacher should lead a five-minute class discussion about different types of flooring that can be used to convert a basement floor into a home office. The discussion should center on what types of flooring are available and characteristics of each flooring type.

The teacher should then distribute copies of the “Flooring Options” table shown below. Explain to the class that this table has the three flooring types they will consider for the home office. This will be used in their response to *Part B*. Point out that the pricing in the table is not always in the same units and that students will need to consider this when using the table in *Part B*.

Flooring Options

Type	Option 1	Option 2	Option 3
Carpet	\$31.98 for box of carpet tiles (20 sq. ft.)	\$8.73 sq. yd.	\$22.23 sq. yd.
Tile	16 in. × 16 in. tiles \$6.49 sq. ft.	12 in. × 12 in. tiles \$1.48 sq. ft.	12 in. × 12 in. tiles \$2.89 sq. ft.
Laminate	\$8.91 sq. yd.	\$1.69 sq. ft.	\$3.99 sq. ft.

Session 1

Home Office

You want to finish your basement and use it as a home office. You plan to hire a contractor to do the work. The amount charged by the contractor is based on the area of the room to be finished. The amount, however, does not include the cost of flooring.

In this task, you will use a blueprint of the basement. You need to decide what area of the basement will be finished by the contractor. You also need to decide on the type of flooring that will go in the basement and price it so that you stay within your budget. Once you know the area of the home office, you will use that information to help determine how it will affect your taxes for the next year.

To accomplish this, you will do the following:

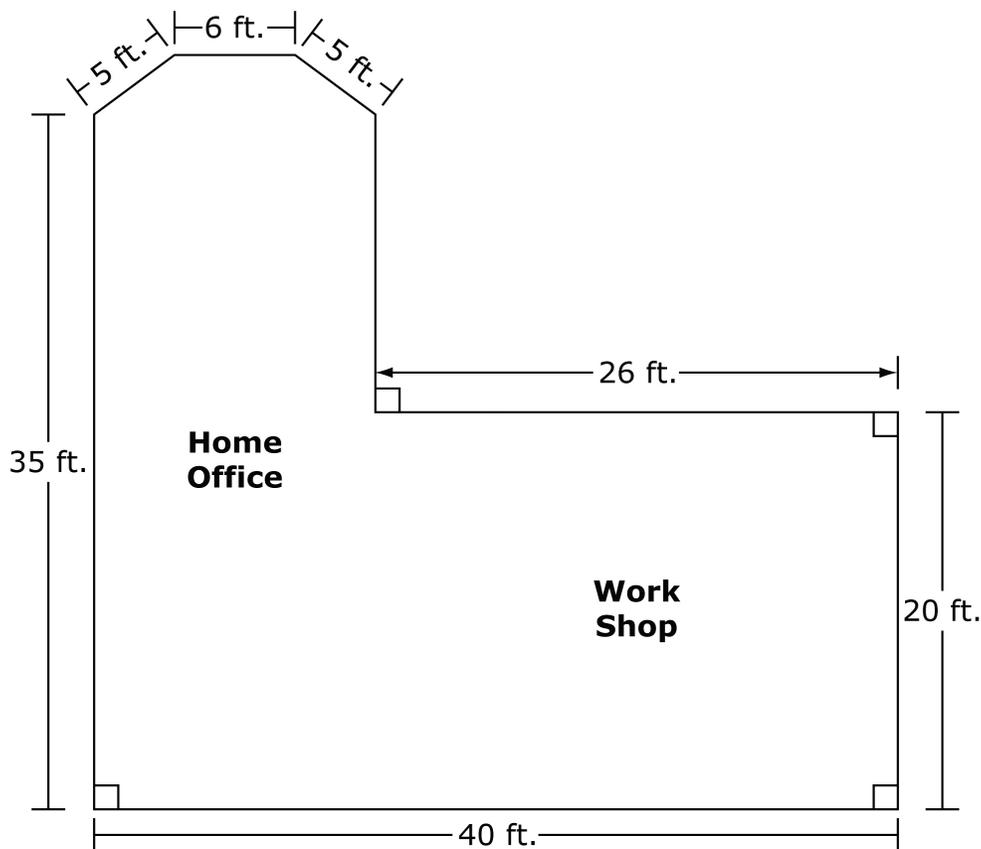
1. Find the maximum area of the basement that can be finished based on the rate charged by the contractor.
2. Decide on the type of flooring you will use given a "Flooring Options" table and calculate the cost of the flooring.
3. Calculate the area used for the actual home business, excluding the area of a bathroom.
4. Calculate expected tax deductions for converting part of the basement as a home office using the following:
 - the area of the home office
 - the area of the entire house
 - the past year's utility expenses

Part A

**Determine the Area
of the Finished Basement**

You want part of your basement converted into a home office. You will hire a contractor to do this work. Your budget for the contractor is \$30,000. The contractor charges \$50 per square foot to finish the basement for the home office. The finished work includes everything but the cost of flooring.

The diagram below represents the blueprint of your basement.



Only the left side of the basement will be finished for the home office. The workshop area will remain unfinished. You want the

greatest possible area for the home office based on your contractor budget.

1. Determine the area of the basement that will be used for the home office. This involves separating the home office area from the workshop area. To do this, draw a vertical line between the two areas directly on the blueprint. The left side of the blueprint will represent the area of the home office. The right side will represent the area of the workshop.

[Use the partition or single line segment TE template or ruler tool.]

Explain how you decided where to draw the line that separates the two areas. In your explanation, give the dimensions of the rectangular workshop.

Part B

Flooring

Use the “Flooring Options” table to help you choose a type of flooring for the finished basement. You will use the average price of that flooring type to calculate your flooring cost.

Base your decision on the following criteria:

- Flooring options in the table
- Average cost of each flooring option
- A flooring budget of \$2000

To find the average cost of each flooring option, use the mean costs of those given in the table. If costs are not in the same units, do the conversions necessary to change them to the same units.

2. What type of flooring did you decide to use? Explain whether or not it was necessary to pick the least expensive flooring option in order to stay within your budget.

3. Based on the flooring budget and the average cost of the flooring you chose, find the total cost of flooring for the finished basement. Show or explain how you found your answer.

End of Session 1

Session 2

Part C

Including a Bathroom

You decide to include a bathroom as part of your home office. You ask the contractor to allow 10% of the finished portion of the basement to be used for the bathroom. You also want the length of the bathroom to be 3 feet longer than the width.

4. Write a polynomial equation that can be used to find the dimensions of the bathroom. Then determine the dimensions of the bathroom. Round each dimension to the nearest half-foot.

Part D

Home Office Expenses and Tax Deductions

When you use a home office, a percentage of total home utility expenses can be deducted from your taxes. This is based on the percentage of your house that is occupied by the home office. When the finished basement is complete, the total square footage of the entire house will include the home office portion of the finished basement but not the bathroom. The area used as the home office will also be used as part of the tax calculation for what you owe to the government.

You want to estimate the amount of utility expenses that can be deducted from next year’s taxes, when your home office is complete. Below is a spreadsheet which lists all of your utility expenses for the past year.

Utility Expenses				
Month	Phone	Heat	Electricity	Trash
Jan	\$61.62	\$95.47	\$152.54	\$316.00
Feb	\$62.71	\$154.83	\$164.89	
Mar	\$57.05	\$124.80	\$125.29	
Apr	\$67.48	\$75.57	\$130.03	Sewer
May	\$66.47	\$33.80	\$192.15	\$94.94
Jun	\$56.23	\$32.30	\$235.05	
Jul	\$50.13	\$35.22	\$254.02	
Aug	\$78.52	\$36.42	\$250.13	Water
Sep	\$66.33	\$31.41	\$240.34	\$178.33
Oct	\$57.73	\$31.38	\$177.52	
Nov	\$57.97	\$54.06	\$229.53	
Dec	\$61.52	\$172.54	\$196.26	
Totals:	\$743.76	\$877.80	\$2,347.75	\$589.27

A percentage of these total expenses can be applied as a tax deduction. However, the phone expenses include some personal calls that cannot be deducted. Only the monthly phone fee and your business-related calls can be deducted.

Your phone bill contains these charges:

- \$36 monthly fee for using the phone
- \$0.12 per minute for each long-distance call made

Your business-related calls are all long-distance calls. You kept a record of the number of minutes you were charged for business-related calls last year. The spreadsheet below shows these data by month.

Business Calls		
Month	Number of Minutes	Phone Expenses
Jan	96	
Feb	68	
Mar	90	
Apr	165	
May	129	
Jun	99	
Jul	48	
Aug	204	
Sep	155	
Oct	67	
Nov	80	
Dec	52	
Total:		

[Note: Students should be able to use the above spreadsheet to enter formulas and calculate costs in any of the empty cells.]

5. Determine a function to find the total phone expense that can be applied toward your home office tax deduction. Apply the function in the spreadsheet to calculate each month's phone expense that applies toward the tax deduction. Find the total applicable phone expense. Use the labeled column in the spreadsheet to show these amounts.

You are now ready to estimate the total amount of utility expenses that can be applied toward your tax deduction next year. Your estimate should account for the following:

- The square footage of your house before adding the home office is 1850 square feet.
 - The areas of bathrooms are not included in the square footage of the house.
 - Deductions are based on the total of last year's utility expenses. These include heat, electricity, trash, water, sewer, and business-related phone expenses.
 - The expected increases in utility expenses are due to the use of the home office.
6. Determine a reasonable estimate for the amount of the tax deduction for next year's taxes. Show or explain how you determined this estimate. You may use a combination of diagrams, mathematical equations or formulas, and words.

End of Session 2

Sample Top-Score Response:

Sample table prepared by teacher:

Flooring Options

Type	Option 1	Option 2	Option 3
Carpet	\$31.98 for box of carpet tiles (20 sq. ft.)	\$8.73 sq. yd.	\$22.23 sq. yd.
Tile	16 in. × 16 in. tiles \$6.49 sq. ft.	12 in. × 12 in. tiles \$1.48 sq. ft.	12 in. × 12 in. tiles \$2.89 sq. ft.
Laminate	\$8.91 sq. yd.	\$1.69 sq. ft.	\$3.99 sq. ft.

Part A

1. The student should provide a complete and correct explanation of how the calculation for the area of the home office was made. The separator line should be marked so that the area of the workshop is 22 feet long and 20 feet wide.

First I calculated the largest possible area of the home office based on the contractor budget.

Largest possible area of home office: $\$30,000 \div \$50 \text{ per sq ft} = 600 \text{ total sq ft}$

Then I divided the blueprint into smaller shapes and found the areas of those shapes. The top trapezoid is made up of two 3-4-5 right triangles and one 3x6 rectangle.

Its height is 3 feet, so its area is $\frac{1}{2}(6 + 14)(3) = 30 \text{ sq ft}$.

The area of the large rectangle below the trapezoid is $(14)(35) = 490 \text{ sq ft}$.

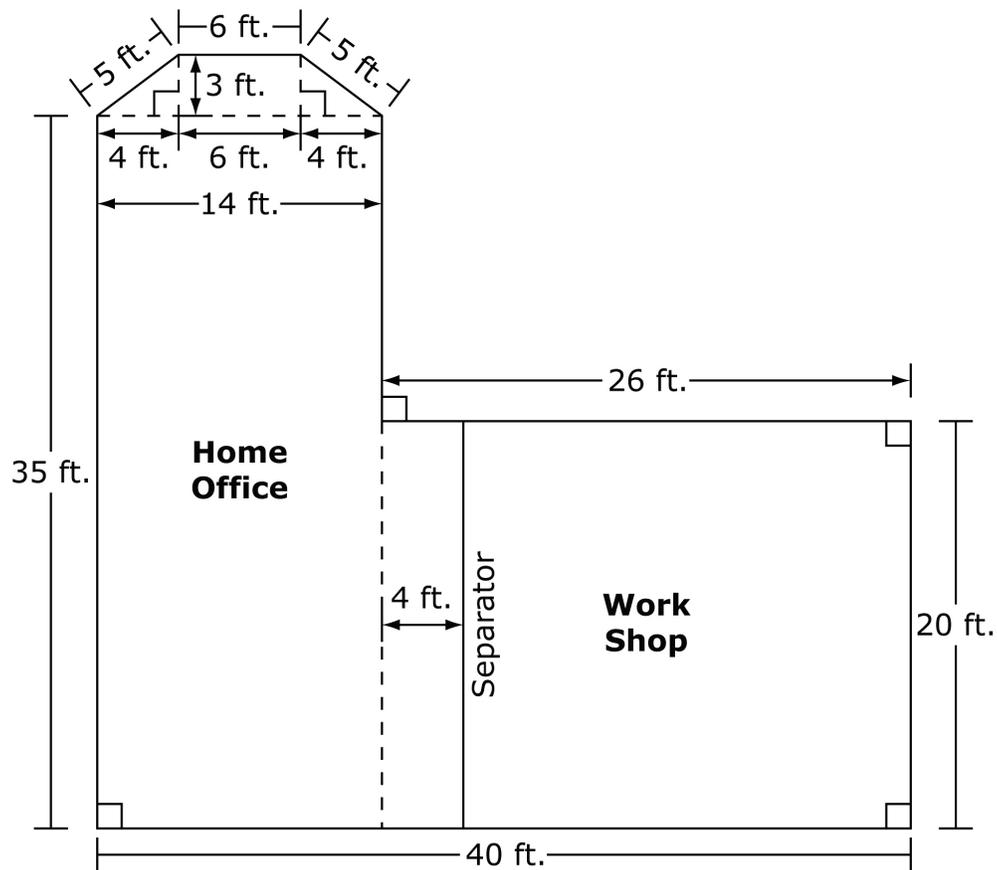
To see how far I must go into the section near the workshop, I need to solve this equation:

$$600 = 30 + 490 + (20x)$$

$$600 - 520 = 20x$$

$$x = 4$$

The home office extends 4 feet toward the workshop to give it a total area of 600 sq ft.



The width of the workshop is 20 feet. The length of the workshop is $26 - 4 = 22$ feet. Its dimensions are 20 ft. by 22 ft. Its area is $(20)(22) = 440$ sq ft.

Part B

2. The student should choose one type of flooring for the home office from the "Flooring Options" table. The student should compare the average flooring costs in the table. The average costs should reflect any converted costs if the units in their pricing were not the same. The student should explain how the cost relates to the budget and whether or not he/she had to choose the least expensive flooring option in order to stay within his or her budget.

The average cost of each flooring type is:

Carpeting average:

Option 1: $\$31.98 \div 20 = \1.60 sq ft

Option 2: $\$8.73 \text{ sq yd} \times \frac{1 \text{ sq yd}}{9 \text{ sq ft}} = \0.97 sq ft

Option 3: $\$22.23 \text{ sq yd} \times \frac{1 \text{ sq yd}}{9 \text{ sq ft}} = \2.47 sq ft

Average = $(\$1.60 + \$0.97 + \$2.47) \div 3 = \1.68 sq ft

Tile average:

Option 1: \$6.49 sq ft

Option 2: \$1.48 sq ft

Option 3: \$2.89 sq ft

Average = $(\$6.49 + \$1.48 + \$2.89) \div 3 = \3.62 sq ft

Laminate average:

Option 1: $\$8.91 \text{ sq yd} \times \frac{1 \text{ sq yd}}{9 \text{ sq ft}} = \0.99 sq ft

Option 2: \$1.69 sq ft

Option 3: \$3.99 sq ft

Average = $(\$0.99 + \$1.69 + \$3.99) \div 3 = \2.22 sq ft

The flooring budget, in cost per sq ft, is $\$2000 \div 600 \text{ sq ft} = \3.33 sq ft.

Tile costs too much for the given budget, so I chose the laminate flooring. The laminate costs more than carpeting, on average, but the total cost still fits within the budget.

3. The student should calculate the cost of flooring based on the square footage determined in *Part A* and the average cost of the flooring determined in question 2. The calculation should include any conversions of flooring costs to square feet. The total cost should also remain within the budget amount of \$2000.

Budget: \$2000

Average cost per sq ft of laminate: \$2.22 sq ft

Area of flooring: 600 sq ft

Cost of flooring = $600 \times \$2.22 = \1332

Part C

4. The student writes the correct polynomial equation.

$$w(w + 3) = 600(10\%)$$

$$w^2 + 3w = 60$$

$$w^2 + 3w - 60 = 0$$

The student correctly solves the equation for w using the quadratic formula and rounds the answer to the nearest half-foot.

$$w = \frac{-3 \pm \sqrt{3^2 - 4(1)(-60)}}{2(1)} = \frac{-3 \pm \sqrt{9 + 240}}{2} \approx \frac{-3 \pm 15.7797}{2}$$

$$w \approx 6.39 \text{ or } -9.39$$

Since widths cannot be negative, the width ≈ 6.39 ft which, to the nearest half-foot, rounds to 6.5 ft. The length to the nearest half-foot = $6.5 + 3 = 9.5$ ft.

Part D

5. The student correctly determines a function for the phone expense, $y = 0.12x + 36$, and

applies it to the data in the spreadsheet. The student writes the function as a formula in one of the empty columns of the spreadsheet and finds the sum of all 12 months of resulting data.

Business Calls		
Month	Number of Minutes	Phone Expenses
Jan	96	\$47.52
Feb	68	\$44.16
Mar	90	\$46.80
Apr	165	\$55.80
May	129	\$51.48
Jun	99	\$47.88
Jul	48	\$41.76
Aug	204	\$60.48
Sep	155	\$54.60
Oct	67	\$44.04
Nov	80	\$45.60
Dec	52	\$42.24
Total:		\$582.36

6. Student work and explanations should include the following:

The student finds the correct area of the home office, not including the bathroom.

Bathroom area is 10% of the home office area = $600 \times 10\% = 60$ sq ft.

Area of home office, excluding bathroom = $600 - 60 = 540$ sq ft

The student finds the correct percent of square footage of the home office.

$540 \div (1850 + 540) \approx 0.2259$ or 22.59%. To the nearest percent, the home office is 23% of the total square footage.

The student finds the correct applicable utility expenses from last year's data.

heat + electricity + trash + sewer + water + phone

$$877.80 + 2347.75 + 316 + 94.94 + 178.33 + 582.36 = \$4397.18$$

The student reasonably identifies utility expenses that are likely to increase and estimates the increase in expenses due to the use of the home office.

Not all utility expenses will change. Certain ones are not affected by an increase in the square footage of a house, such as phone, trash, sewer, and water expenses. Others will increase, however, with additional square footage. The utilities that are expected to increase are heat and electricity, since the home office will need to be climate-controlled and powered for light and business equipment.

To estimate the amount that heat and electricity expenses are expected to increase, I'll estimate the amount of these utility expenses for 540 square feet of the existing house. Then I'll add that increased amount to the total expenses.

$$(540 \div 1850)(877.80 + 2347.75) = \$941.51$$

$$\text{Total estimated expenses with the home office} = 4397.18 + 941.51 = \$5338.69$$

The amount of the tax deduction =

$$\text{percent of total area occupied by home office} \times \text{total estimated expenses with home office}$$

$$23\% \times 5338.69 = \mathbf{\$1227.90}$$

Scoring Notes:

Class discussion prior to starting the task will not be scored. Each question in *Parts A* through *D* is evaluated individually. The total number of points is determined by adding the points assigned for each question.

Scoring Rubric:

Scoring Rubric for **Part A:**

Question 1: Responses to this item will receive 0–4 points, based on the following:

4 points: The student shows a thorough understanding of how to use geometric and algebraic concepts to determine area. The student correctly identifies where to place the partition line in the basement. The student correctly calculates areas for sections of the home office using the Pythagorean theorem and algebraic equations. The student correctly determines the dimensions of the workshop area.

3 points: The student shows a strong understanding of how to use geometric and algebraic concepts to determine area. The student correctly identifies where to place the partition line in the basement. The student correctly calculates areas for sections of the home office using the Pythagorean theorem and algebraic equations. However, the student either forgets to determine the dimensions of the workshop or calculates incorrect dimensions. **OR** The student correctly identifies where to place the partition line in the basement and correctly determines the dimension of the workshop area. However, the student's work or explanation is incomplete.

2 points: The student shows partial understanding of how to use geometric and algebraic

concepts to determine area. The student makes one or two calculation errors for the area used for the home office. As a result, the student identifies a partition line in the basement close to, but not exactly, where it should go. The student determines the correct dimensions of the workshop based on the incorrect partition line.

1 point: The student shows a limited understanding of how to use geometric and algebraic concepts to determine area. The student makes some appropriate area calculations to help determine where to place the partition line but does not identify the correct location for the partition. The student does not determine the correct dimensions of the workshop.

0 points: The student shows inconsistent understanding of how to use geometric and algebraic concepts to determine area. The student's work contains many calculation errors, a missing or incorrect partition line, and missing or incorrect dimensions of the workshop.

Scoring Rubric for **Part B**:

Question 2: Responses to this item will receive 0–2 points, based on the following:

2 points: The student shows a thorough understanding of how to analyze real-world scenarios and make productive use of knowledge to make decisions and solve problems. The student finds the correct average costs of each flooring type from the "Flooring Options" table, including any necessary conversions. The student refers to staying within the budget and whether or not he/she needed to choose the least expensive flooring option.

1 point: The student shows partial understanding of how to analyze real-world scenarios and make productive use of knowledge to make decisions and solve problems. The student finds the average costs of each flooring type from the "Flooring Options" table, including any necessary conversions. However, the student does not refer to the budget as a reason for choosing the type of flooring or whether or not he/she needed to choose the least expensive flooring option. **OR** The student makes one or two errors calculating the average costs of each flooring type from the "Flooring Options" table, including errors in any necessary conversions. However, the student stays within the budget and refers to whether or not he/she needed to choose the least expensive flooring option.

0 points: The student shows inconsistent understanding of how to analyze real-world scenarios and make productive use of knowledge to make decisions and solve problems. The student incorrectly calculates the average costs of one or more flooring types and does not stay within the budget or refer to whether or not he/she needed to choose the least expensive flooring option.

Question 3: Responses to this item will receive 0–1 point, based on the following:

1 point: The student shows a thorough understanding of how to solve problems in applied math. The student correctly calculates the cost of the flooring based on the cost of the flooring and square footage of the area. The student makes any necessary conversions and stays within the specified budget.

0 points: The student shows inconsistent understanding of how to solve problems in

applied math. The student incorrectly calculates the cost of the flooring. The student does not make necessary conversions and/or does not stay within the specified budget.

Scoring Rubric for **Part C:**

Question 4: Responses to this item will receive 0–3 points, based on the following:

3 points: The student shows a thorough understanding of how to apply algebraic concepts to solve problems in applied math. The student correctly writes and solves a polynomial equation to find the dimensions of the bathroom, rounded to the nearest half-foot.

2 points: The student shows some understanding of how to apply algebraic concepts to solve problems in applied math. The student correctly writes a polynomial equation but makes a minor error applying the quadratic formula.

1 point: The student shows partial understanding of how to apply algebraic concepts to solve problems in applied math. The student correctly writes a polynomial equation but makes one or two errors solving the equation. The student also does not round the dimensions to the nearest half-foot. **OR** The student incorrectly writes a polynomial equation but solves that equation correctly for the dimensions of the bathroom, rounded to the nearest half-foot.

0 points: The student shows inconsistent understanding of how to apply algebraic concepts to solve problems in applied math. The student does not correctly write or solve a quadratic equation to find the dimensions of the bathroom.

Scoring Rubric for **Part D:**

Question 5: Responses to this item will receive 0–2 points, based on the following:

2 points: The student shows a thorough understanding of how to apply an algebraic function and spreadsheet technology to solve problems in applied math. The student enters a correct function into the spreadsheet and determines the total deductible phone expenses.

1 point: The student shows partial understanding of how to apply an algebraic function and spreadsheet technology to solve problems in applied math. The student enters into the spreadsheet a function with the slope and the y -intercept reversed. However, the student applies this formula to each month and sums the amounts to determine the total. **OR** The student enters a correct function into the spreadsheet to determine each month's deductible phone expense but does not find the total sum.

0 points: The student shows inconsistent understanding of how to apply an algebraic function and spreadsheet technology to solve problems in applied math. The student enters an incorrect function into the spreadsheet unrelated to reversing the slope and y -intercept.

Question 6: Responses to this item will receive 0–4 points, based on the following:

4 points: The student shows a thorough understanding of how to analyze complex, real-world scenarios and construct mathematical models to solve problems. The student

calculates the correct percent of square footage occupied by the home office, excluding the area of the bathroom. The student determines the correct applicable utility expenses from the prior year. The student provides a reasonable explanation as to which utilities might increase with the addition of the home office and determines a reasonable estimate for those increased amounts. The student determines a reasonable estimate of the amount of the tax deduction based on the percentage of home office square footage.

3 points: The student shows a strong understanding of how to analyze complex, real-world scenarios and construct mathematical models to solve problems. The student correctly determines most of the calculations and/or estimates needed to assess the amount of the tax deduction for utility expenses.

2 points: The student shows partial understanding of how to analyze complex, real-world scenarios and construct mathematical models to solve problems. The student correctly determines some of the calculations and/or estimates needed to assess the amount of the tax deduction for utility expenses.

1 point: The student shows a limited understanding of how to analyze complex, real-world scenarios and construct mathematical models to solve problems. The student correctly determines one of the calculations and/or estimates needed to assess the amount of the tax deduction for utility expenses.

0 points: The student shows inconsistent understanding of how to analyze complex, real-world scenarios and construct mathematical models to solve problems. The student does not correctly determine any of the calculations or estimates needed to assess the amount of the tax deduction for utility expenses.