

Cubism Meets Polyhedra

Big Ideas
<p>Unit of Instruction</p> <p>Cubism meets Polyhedra</p>
<p>Geometry Concept</p> <ul style="list-style-type: none"> • Platonic and Archimedean solids in two-and three dimensions
<p>Rationale</p> <p>This project is rich in geometry/mathematics terminology. It focuses on visualizing three-dimensional objects, emphasizing the transition from two-dimensional to three-dimensional figures. It also encourages creativity.</p>
<p>NCTM 9-12 Standards</p> <ul style="list-style-type: none"> • Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships. • Use visualization, spatial reasoning, and geometric modeling to solve problems. • Make and investigate mathematical conjectures. • Build new mathematical knowledge through problem solving. • Recognize and use connections among mathematical ideas. • Use representations to model and interpret physical, social, and mathematical phenomena.
<p>Idaho Content Standards</p> <ul style="list-style-type: none"> • G.2.2.1 Understand and use formulas to calculate the perimeter, circumference, area, surface area, and volume of geometric figures. • G.4.1.1 Analyze properties and determine attributes of two- and three-dimensional objects. • G.4.1.2 Explore congruence and similarity among classes of two-dimensional objects and solve problems involving them. • G.4.4.1 Draw and construct representations of two-dimensional geometric objects using a variety of tools. • 9-12.VA.2.1.1 Develop and present basic analyses of works of visual art from structural, historical, and cultural perspectives. • 9-12.VA.3.2.1 Choose purposefully between visual characteristics of a variety of media and use these to communicate one's own idea.

Background	
Vocabulary	
Archimedean Solid	Octahedron
Blind Contour	Pattern
Contour	Platonic Solid
Cubism	Polygons
Dodecahedron	(equilateral triangles, squares, regular pentagons, regular hexagons, regular octagons, regular decagons)
Edge	Polyhedra
Face	Regular Polygon
Facet	Rhombus
Form	Shape
Frustrum	Snub
Hexahedron	Tetrahedron
Icosahedron	Truncate
Monochromatic	Vertex
Neutral	
Math Instruction (pre- or post-project)	
Investigating key attributes of regular polygons and polyhedra	

Driving Question

Project Objective

- Students will use regular polygons to create Platonic and Archimedean three-dimensional solids.
- Working in groups, students will create a three-dimensional Cubist sculpture utilizing Platonic and Archimedean solids.
- **Extension**
- Students will create a two-dimensional artwork from the three-dimensional Cubist sculpture.
- Students will analyze the following questions: In terms of Cubism, what is face? How can it be defined?

Questions to be Answered

What are the differences/similarities of Platonic and Archimedean solids?

What are faces, edges, vertices of a shape?

What is a facet?

How many of each of the regular polygons are required to make each of the solids?

Which shapes intersect at each of the vertices?

Find and explain the meaning of the roots for each shape name. (e.g. Icosahedron = Icosa + Hedron= 20+ faces.)

Note: answers are attached as a supplement (see page 12).

What is Cubism?

How does Cubism relate to geometric shapes?

What is the difference between a shape and a form?

Can you identify Platonic solids in Cubist work? (show examples of Picasso)

Can you identify Archimedean solids in Cubist work? (show examples of Picasso)

What is a monochromatic color scheme?

What is a neutral?

Note: information on cubism is included as a supplement (see page 13) or check the art glossary in the back of the curriculum.

Materials

Materials Required

- Card stock (colors if possible)
- Rubber bands size 12 or 14 (this size is important)
- Patterns
- Colored pencils, markers, crayons
- Wrapping paper
- Scissors
- Hole punch
- Hot glue
- White glue
- Paint (optional)
- Recycled greeting cards (optional)
- Decorative materials (beads, twigs, rope, yarn, wrapping paper, sand paper, brown paper sacks, etc.)

Reference Materials

Green, Bruce Seth. *The Cubist Epoch*. Universal Education and Visual Artists. KVC Home Video, 1971.

Band-it Polyhedra Kit #1, Platonic and Archimedean Solids, Laron Thomas Smith. Student examples.

Photo examples.

www.scienceu.com/geometry/facts/solids/handson.html

www.georgehart.com/virtual-polyhedra/archimedean-info.html

Schwartzman, Steven. *The Words of Mathematics: An Etymological Dictionary of Mathematical Terms Used in English*. MAA, 1994

Lesson Outline

Description of Activity

- Introduce Platonic and Archimedean solids. (45 min.)
- Choose one Platonic and one Archimedean solid to create. (5 min.)
- Using patterns from the polyhedra kit students will cut out necessary pieces. (60 - 90 min.)
- Students will decorate the pieces following a theme. They will use the rubber-bands to hold the individual pieces together to create the solids. (60 min. - 90 min.)
- Students will identify and name each of their solids. (10 min.)
- Class will be broken into groups. Groups must be no less than four and no more than 6. Students will view Cubism works and discuss geometric shapes visible within each Cubist work. (15. min.)
- Students will identify possible Platonic and Archimedean solids within each of the Cubist works. (20 min.)

Extension Activity

- Students will complete blind contour drawings.
- Working in groups, students will create a Cubist face using one Archimedean solid as the base (skull) for the face and various Platonic solids to create the facial features. The instructor will show examples. (120 min.)
- Students will decorate their Cubist faces. Students will rename their polyhedra Cubist sculpture that is a result of many polyhedra forms. Students will present to the teacher and class. Class discussion will follow. The artwork will be displayed. (120 min.)
- Extend the assignment by creating individual two-dimensional artworks from the three-dimensional sculptures of the Cubist faces. (180 min.)

Day One

- Introduce Platonic and Archimedean solids (polyhedra) including similarities and differences, history, and other relationships.
- Students will be given a handout on Archimedean and Platonic solids from which they will choose one of each to create.
- Students will begin to cut out shapes from card stock using the patterns given

Alternate Activity:

Pre-made kits of platonic and Archimedean solids are made and stored in plastic bags. Students can make and disassemble shapes.

Day Two

- Continue cutting and decorating the pieces of card stock to be assembled into polyhedra.
- Use rubber bands to connect the pieces to create the polyhedra.
- Have students identify names and types of polyhedra and present to the instructor and/or to the class.
- Students will view Cubist prints.

Day Three

- Students will identify possible polyhedra within each of the Cubist works.
- Class will discuss possible polyhedra shapes within a human face.
- Working in groups students will create a Cubist face utilizing polyhedra forms. (Students may want to create a face that has a theme...i.e. a punk rocker, a baby, a surrealistic face, an alien, etc.)
- Students will decorate their Cubist face utilizing various materials.
- Class presentation and discussion regarding the polyhedra forms present in each Cubist sculpture will occur.

At Home Project

This assignment will be introduced on day three but will continue at home for approximately three days to a week.

Optional Extension:

- Students will create a two-dimensional artwork from the three-dimensional sculpture. Any medium of choice may be used.
- Students will look for patterns in the number of faces, edges, and vertices for each polyhedra.
- Investigate relationships between the number of faces, edges and vertices. This is known as Euler's law. The formula is $Faces = Edges - Vertices + 2$.

Assessment

Rubric

There are two projects: the individual project of the polyhedra and the group project of the Cubist sculpture from polyhedra forms. Projects can be worth any amount of points. A possible rubric is listed below.

Polyhedra-individual project

Creativity: 20 pts.

- Pattern, design, color choices, adornment, materials, assemblage of various materials to enhance design.

Neatness: 20 pts.

- Clean and even edges.
- Neatness of decorations.
- Balanced designs.

Accuracy: 60 pts.

- Pieces cut and assembled accurately.
- Proper identification of type and name of each polyhedra.

Cubism-group project

Creativity: 40 pts.

- Selection of polyhedra forms to create a face.
- Selection of type of face.
- Visual interest. Variety of visual materials.

Neatness: 40 pts.

- Clean and even edges.
- Balanced designs.
- Polyhedra forms fit together properly so that it looks like one unit.

Composition and Collaboration: 40 pts.

- Pieces cut and assembled accurately.
- Individual pieces assembled in an interesting way.
- How well did the group work as a team?

Ideas for Further Independent Student Project

- Cubist sculpture in clay.
- Drawing of Cubist sculpture using any two-dimensional medium.
- Deconstruct any polyhedra and create an interesting geometric design/pattern.
- Festival of Trees-math and art students create a Christmas tree for the Festival of Trees utilizing geometric forms.
- Create a net for the various polyhedra.
- Research the use of polyhedra historically.

Faces of the Archimedean Solids

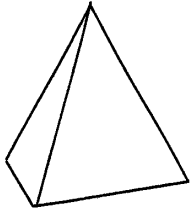
Name of solid	Faces
Truncated Tetrahedron	4 hexagons and 4 triangles
Truncated Cube	6 octagons and 8 triangles
Truncated Octahedron	8 hexagons and 6 squares
Truncated Dodecahedron	12 decagons and 20 triangles
Truncated Icosahedron	20 hexagons and 12 pentagons
Cuboctahedron	6 squares and 8 triangles
Rhombicuboctahedron	18 squares and 8 triangles
Great Rhombicuboctahedron	6 octagons, 8 hexagons, 12 squares
Icosadodecahedron	12 pentagons and 20 triangles
Rhombicosadodecahedron	12 pentagons, 20 squares and 30 triangles
Truncated Icosadodecahedron	12 decagons, 20 hexagons and 30 squares
Snub Icosidodecahedron or Snub Dodecahedron	12 pentagons and 80 triangles
Snub Cube	6 squares and 32 triangles

Directions for creating a Platonic or Archimedean Solid

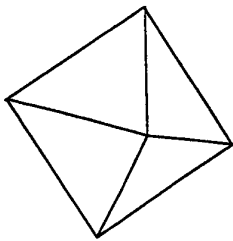
1. Photocopy the polygon patterns onto sheets of card stock.
2. Have students decorate the shapes before they cut them out if possible.
3. Cut out the shapes.
4. Use a straightedge and sharp point (like a compass edge) to lightly score the dotted line of each shape to help the shape bend.
5. Use a hole punch or scissors to cut out the corner of each polygon. This gives you a hook for the rubber band.
6. Fold back at the dotted line.
7. Attach edges of polygons together using size 12 or 14 rubber bands. You may have students research the vertices of the Archimedean solids to find out how to put them together. One excellent site is <http://www.scienceu.com/geometry/facts/solids/handson.html>.

Platonic Solids

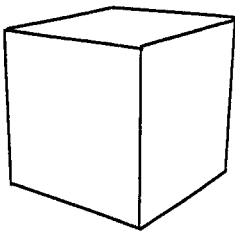
Although Pythagoras was the leader of one of the original groups studying Platonic solids, it was Plato after whom the shapes were named due to his work relating the shapes to atoms, matter and nature. These solids are also called regular polyhedra. Polyhedra is plural for polyhedron. Platonic solids are constructed using the same regular polygon throughout each polyhedron.



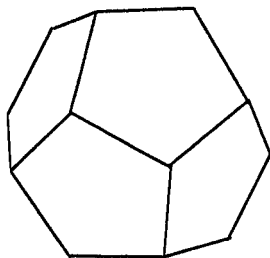
Tetrahedron- made from 4 regular triangles.
Three triangles meet at each of the vertices.



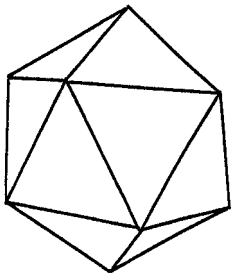
Octahedron- made from 8 regular triangles.
Four of the triangles meet at each vertex of the solid.



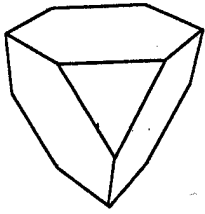
Cube or Hexahedron- made from 6 squares with 3 squares meeting at each vertex.



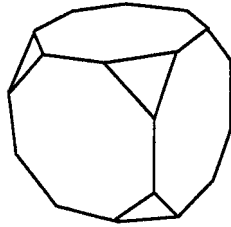
Dodecahedron- made from 12 pentagons with three pentagons meeting at each vertex



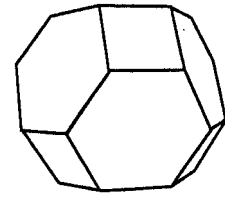
Icosahedron- made from 20 regular (equilateral) triangles with five triangles meeting at each vertex.



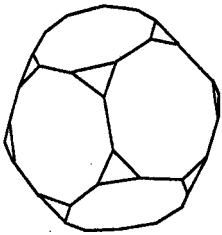
Truncated Tetrahedron
Octahedron



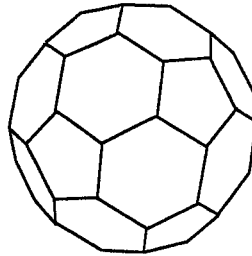
Truncated Cube/Hexahedron



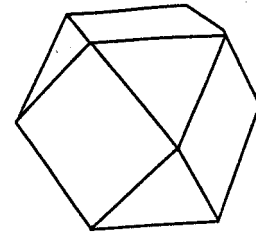
Truncated



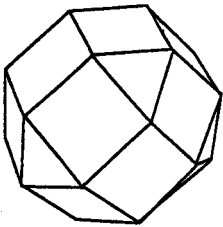
Truncated Dodecahedron



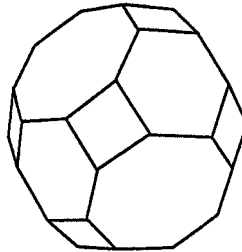
Truncated Icosahedron



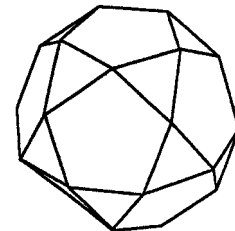
Cuboctahedron



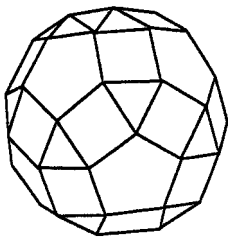
Rhombicuboctahedron



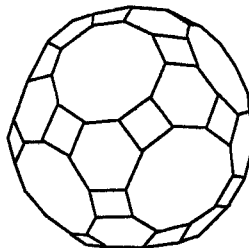
Great Rhombicuboctahedron



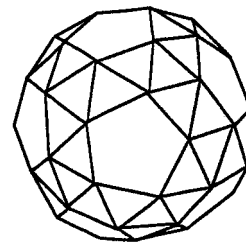
Icosadodecahedron



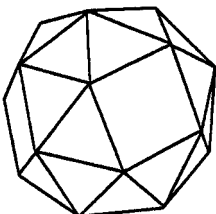
Rhombicosadodecahedron



Truncated Icosadodecahedron
Great Rhombicosadodecahedron



Snub Dodecahedron



Snub Cube

Archimedean Solids

Archimedean Solids consist of faces made from more than one regular polygon shape. These solids are derivations of the Platonic solids.

Cubism Meets Polyhedra: Answers to p. 3

What are the differences/similarities of Platonic and Archimedean solids?

Answer: Each Platonic solid is made from exactly one regular polygonal face. For example, the icosahedron is created using regular triangles while the hexahedron is made out of squares.

What are faces, edges and vertices of a shape? What is a facet?

Answer: A face is the flat polygonal surface of the polyhedron, an edge is the segment where two faces intersect and a vertex is the point of intersection of three or more edges. A facet is a small plane surface (as on a cut gem).

Which polygons and how many of each are required to make each of the solids?

Answer: See attached chart

Which shapes intersect at each of the vertices?

Answer. The notation (3, 6, and 6) means a triangle and 2 hexagons are arranged in cyclic order at each vertex.

Name of solid	Shapes at each vertex
Tetrahedron	3,3,3 (triangle, triangle, triangle)
Octahedron	3,3,3,3
Icosahedron	3,3,3,3,3
Hexahedron	4,4,4
Dodecahedron	5,5,5
Truncated Tetrahedron	3,6,6 (triangle, hexagon, hexagon)
Truncated Cube	3,8,8
Truncated Octahedron	4,6,6
Truncated Dodecahedron	3,10,10
Truncated Icosahedron	5,6,6 (pentagon, hexagon, hexagon)
Cuboctahedron	3,4,3,4
Rhombicuboctahedron	3,4,4,4
Great Rhombicuboctahedron	4,6,8
Icosadodecahedron	3,5,3,5 (triangle, pentagon, triangle, pent)
Rhombicosadodecahedron	3,4,5,4
Truncated Icosadodecahedron	4,6,10
Snub Icosidodecahedron	3,3,3,3,5
Snub Cube	3,3,3,3,4

What are the meanings of the roots for each shape?

Answer: tetra means 4, octa means 8, do means 2, deca means 10, dodeca means 12, ico means 20, hexa means 6, and the words snub and truncate both mean to cut or clip off. Rhomb shows that one set of faces lies in the planes of the rhombic dodecahedron. Others are found in *“The Words of Mathematics: an Etymological Dictionary of Mathematical Terms Used in English,”* by Steven Schwartzman.

Cubism

One of the most influential art movements (1907-1914) of the twentieth century, Cubism was begun by Pablo Picasso (Spanish, 1882-1973) and Georges Braque (French, 1882-1963) in 1907. They were greatly inspired by African sculpture, by painters Paul Cézanne (French, 1839-1906) and Georges Seurat (French, 1859-1891), and by the Fauves.

Cubism began as an intellectual revolt against the artistic expression of previous eras. Among the specific elements abandoned by the cubists were the sensual appeal of paint texture and color, subject matter with emotional charge or mood, the play of light on form, movement, atmosphere, and the illusionism that proceeded from scientifically-based perspective.

The cubists sought to show everyday objects as the mind, not the eye, perceives them-- from all sides at once. The *trompe l'oeil* element of collage and painting was also sometimes used.

In Cubism the subject matter is broken up, analyzed, and reassembled in abstracted form. Picasso and Braque initiated the movement when they followed the advice of Paul Cézanne, who in 1904 said artists should treat "nature" in terms of the cylinder, the sphere, and the cone.

This makes this art form highly conducive to working with geometry and geometric shapes. In art, the term shape refers to an enclosed space defined by other art elements such as line, color, value, and texture. In painting and drawing, shapes may take on the appearance of solid three-dimensional objects, even though they are limited to two dimensions—length and width. Geometric shapes (e.g. circle, oval, polygon, triangle, square) are used at length in cubist works. This two-dimensional character of shape distinguishes it from form, which has depth as well as length and width.

Geometric and Organic Shapes

Geometric Shapes / Rectilinear Shapes

Notice the shapes below and to the left of the red line. Angular shapes therefore we call them **rectilinear shapes**.

Most **rectilinear shapes** can be also called **geometric shapes** however a couple of them are not. These are circles and ovals.

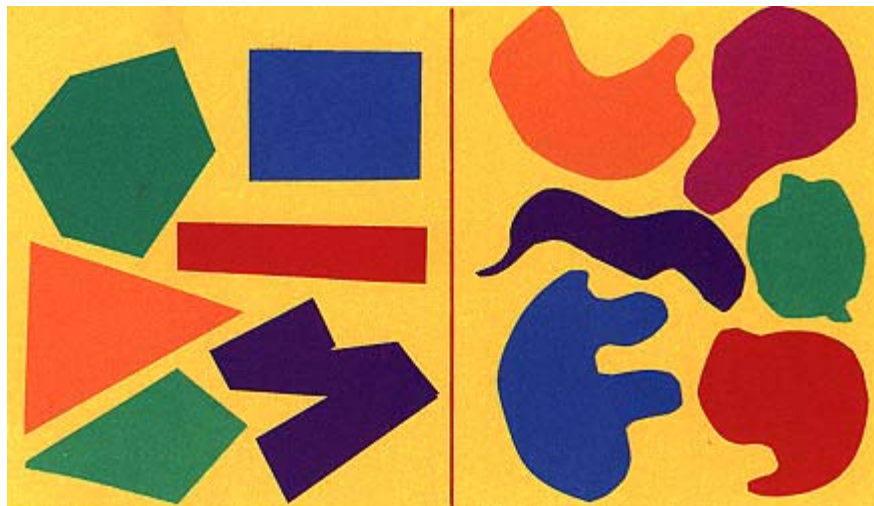
Geometric and rectilinear shapes usually are man-made. A few exceptions to the rule are crystals and honey combs.

Organic Shapes / Curvilinear Shapes

Notice the shapes below and to the right of the red line. They are all curving and flowing therefore we call them **curvilinear shapes**.

Curvilinear shapes could also be called **organic shapes**. We can include circles and ovals.

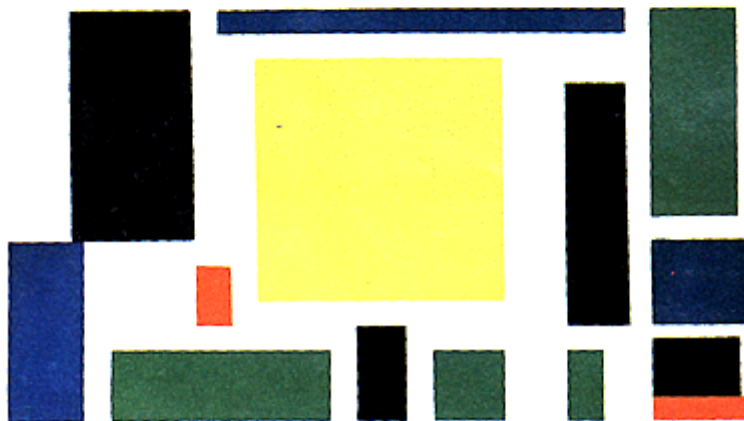
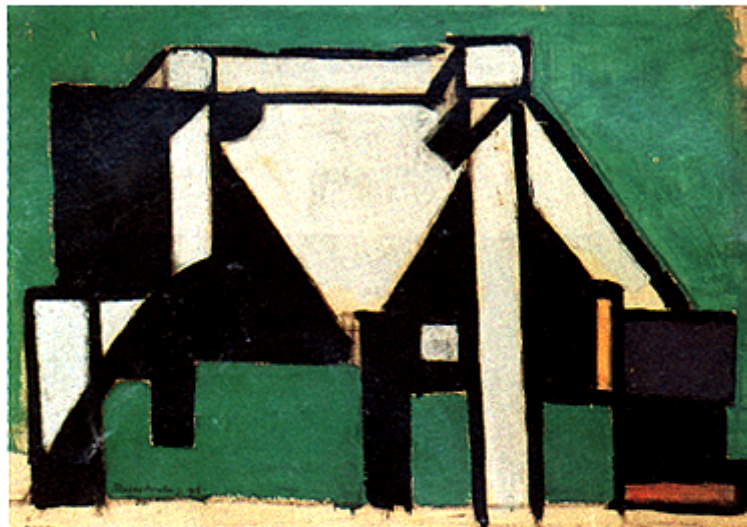
Organic and curvilinear shapes are usually natural shapes. Think of leaves, animal shapes, and plant shapes. Aren't they all organic?



So how would you answer this question?

Circles and ovals are considered to be which? There can be more than one answer.

- Geometric Shapes
- Rectilinear Shapes
- Organic Shapes
- Curvilinear Shapes



"Abstraction of a Cow, Four Stages" by Theo van Doesburg, Museum of Modern Art, NY

- Notice this three-dimensional organic form of the cow in the first image.
- The cow becomes an abstract cow because it becomes geometric but is still 3-D looking.
- Then it becomes flat two-dimensional shapes that one can still see as an abstract cow.
- The fourth stage is simply geometric shapes inspired by a cow.

CUBISM MEETS POLYHEDRA

PLATONIC AND ARCHIMEDEAN SOLIDS



Cubist Face in Three-Dimension



Three-Dimensional Cubist Sculpture

- Cubism is a form of abstraction that concentrates on the breakdown of forms from three-dimensional forms into two dimensional shapes
- Students struggle creating the illusion of three dimensional forms on a flat two-dimensional surface
- The objective of this assignment is to build greater understanding of how polyhedra forms come together to create a complete sculpture



Two-Dimensional Cubist Drawing

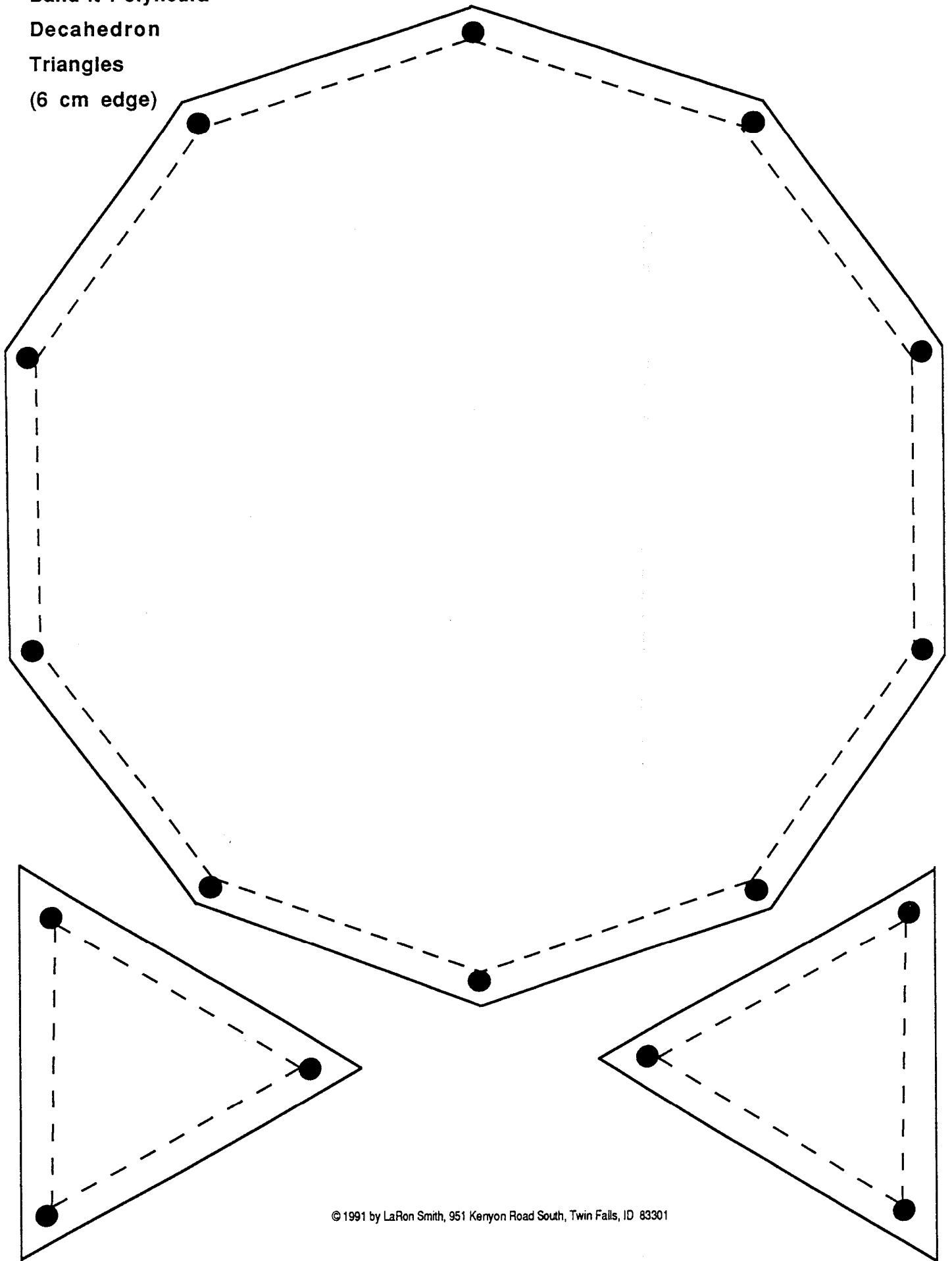


Band-it Polyhedra

Decahedron

Triangles

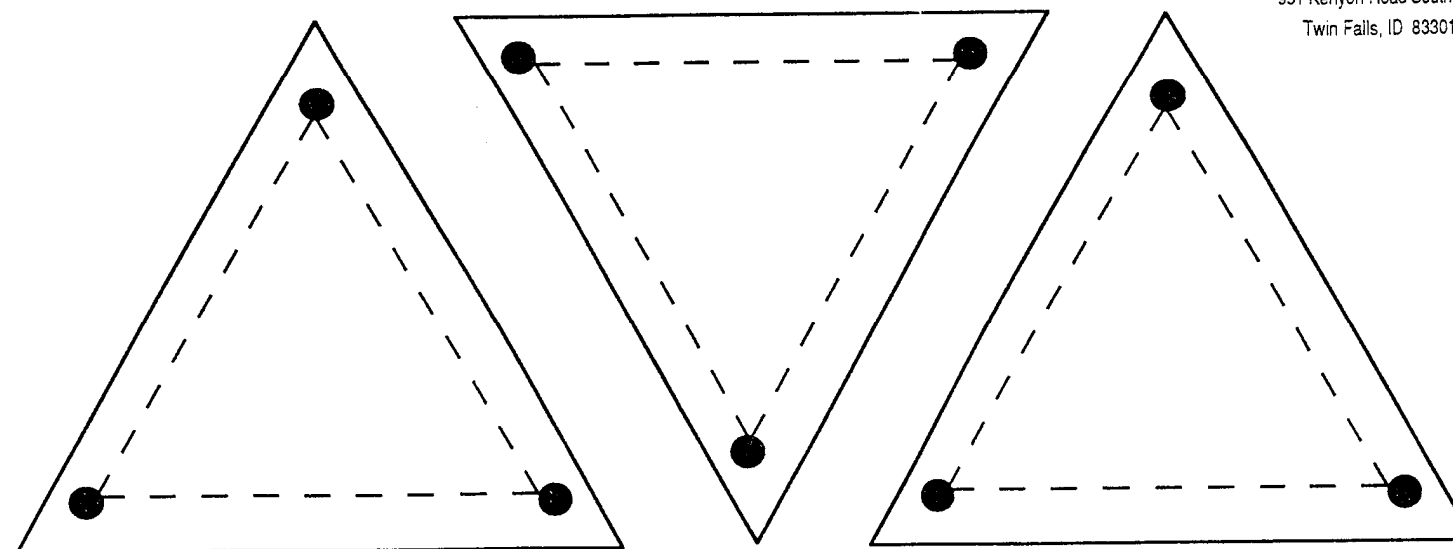
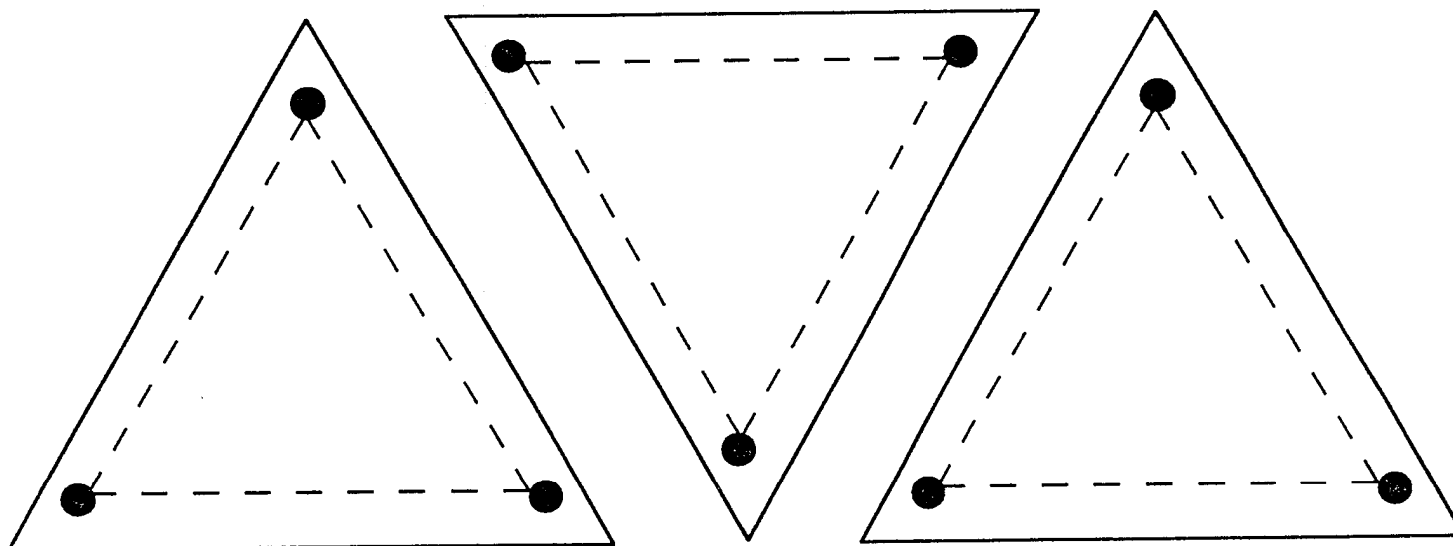
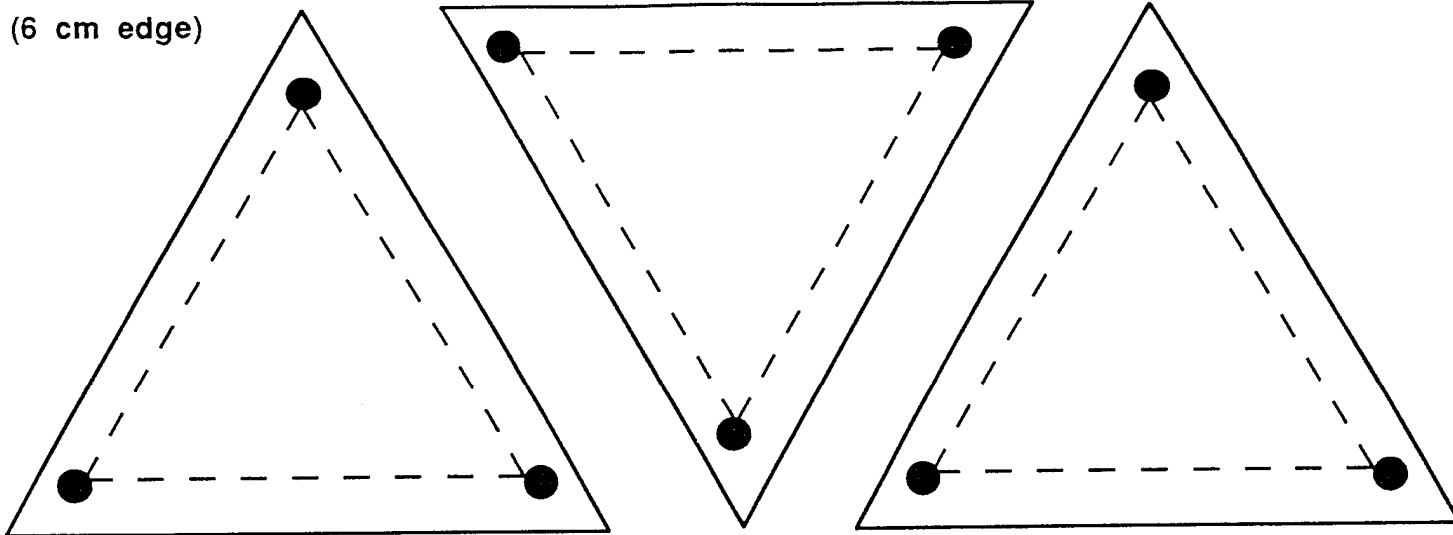
(6 cm edge)



Band-it Polyhedra

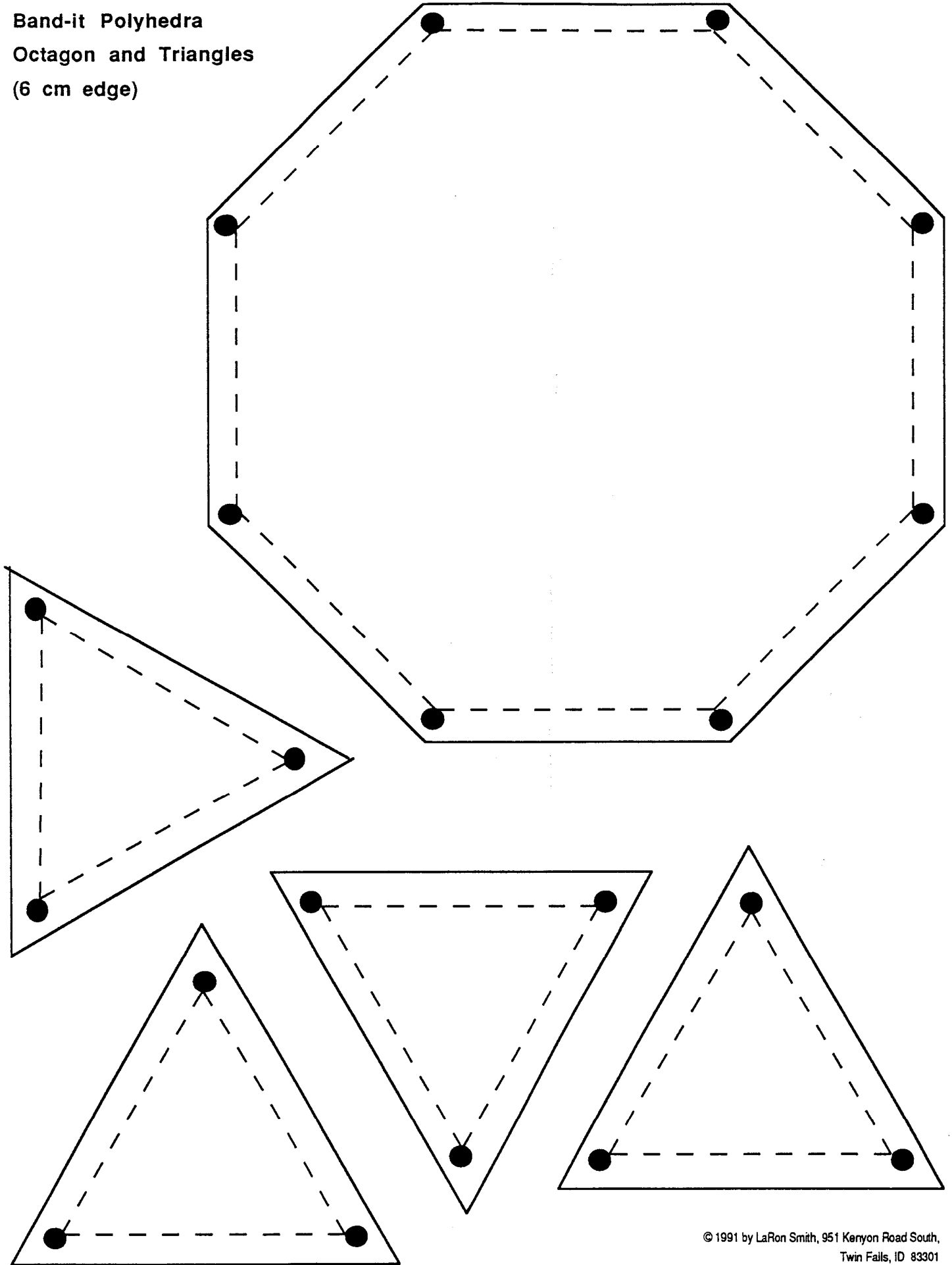
Triangles

(6 cm edge)



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Twin Falls, ID 83301

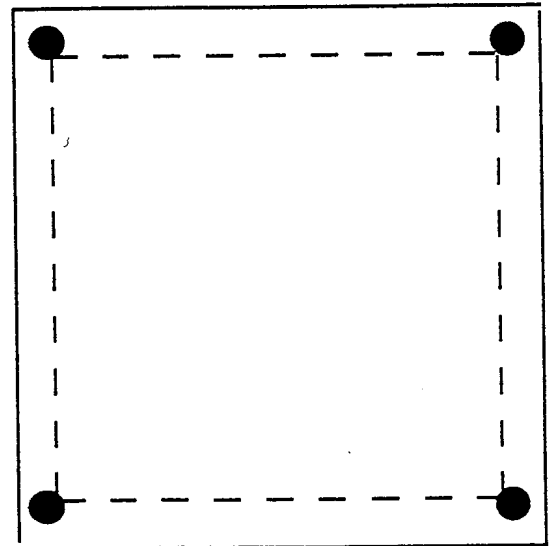
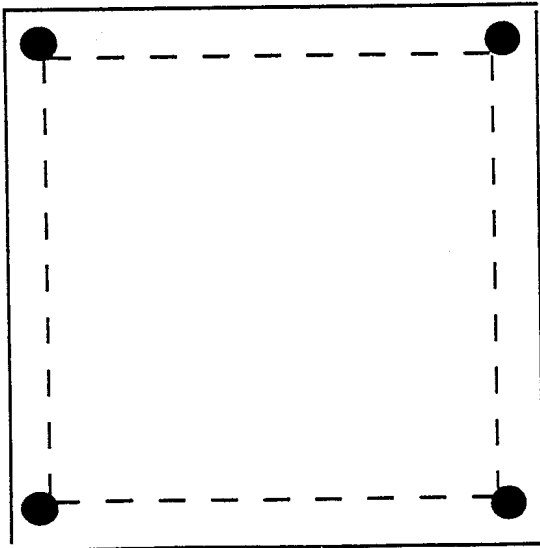
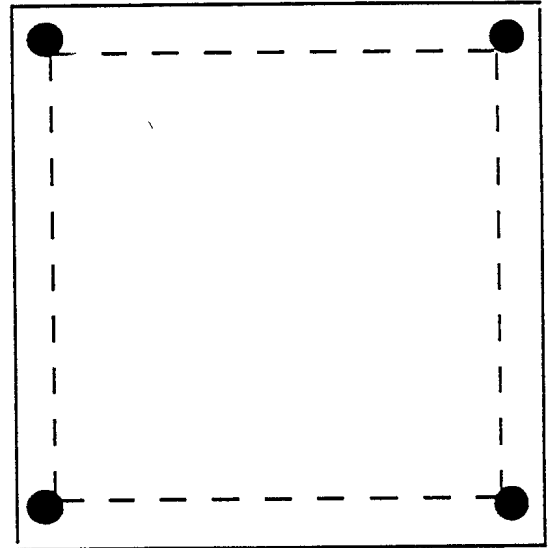
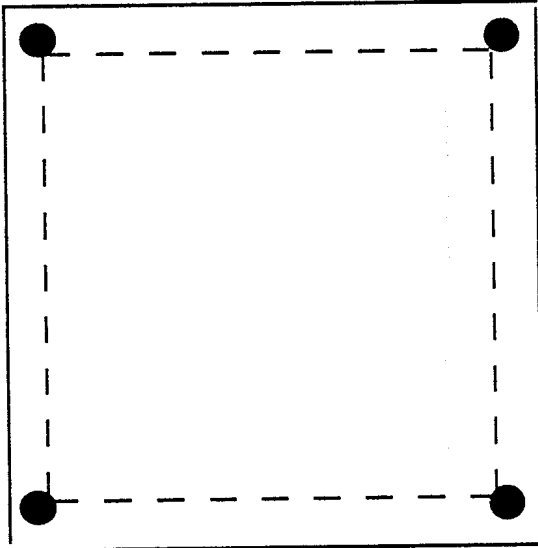
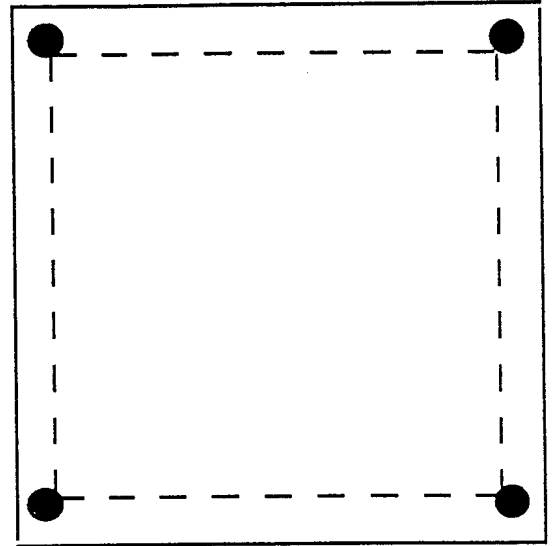
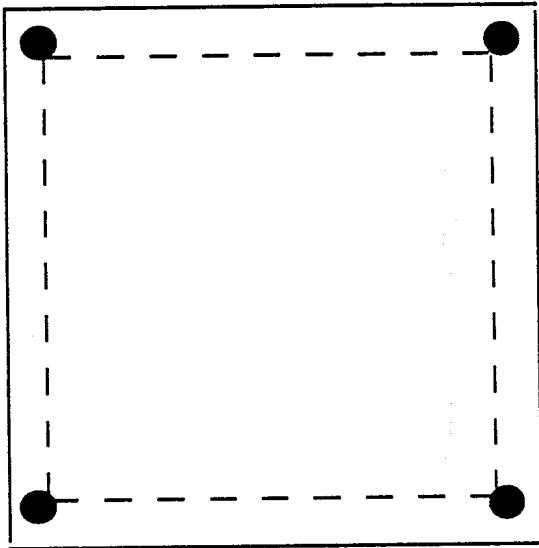
Band-it Polyhedra
Octagon and Triangles
(6 cm edge)



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Band-it Polyhedra

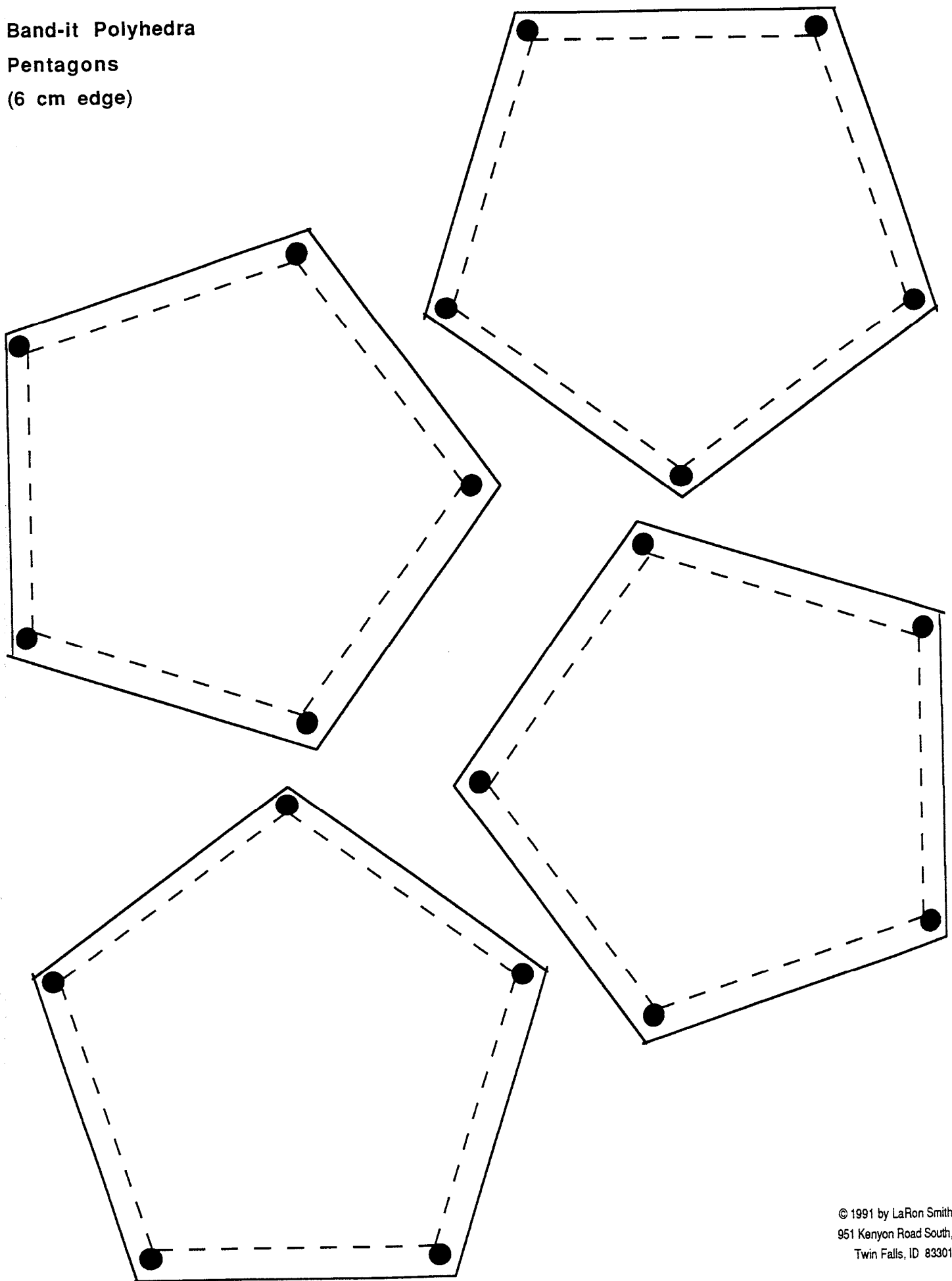
Squares (6 cm edge)



Band-it Polyhedra

Pentagons

(6 cm edge)

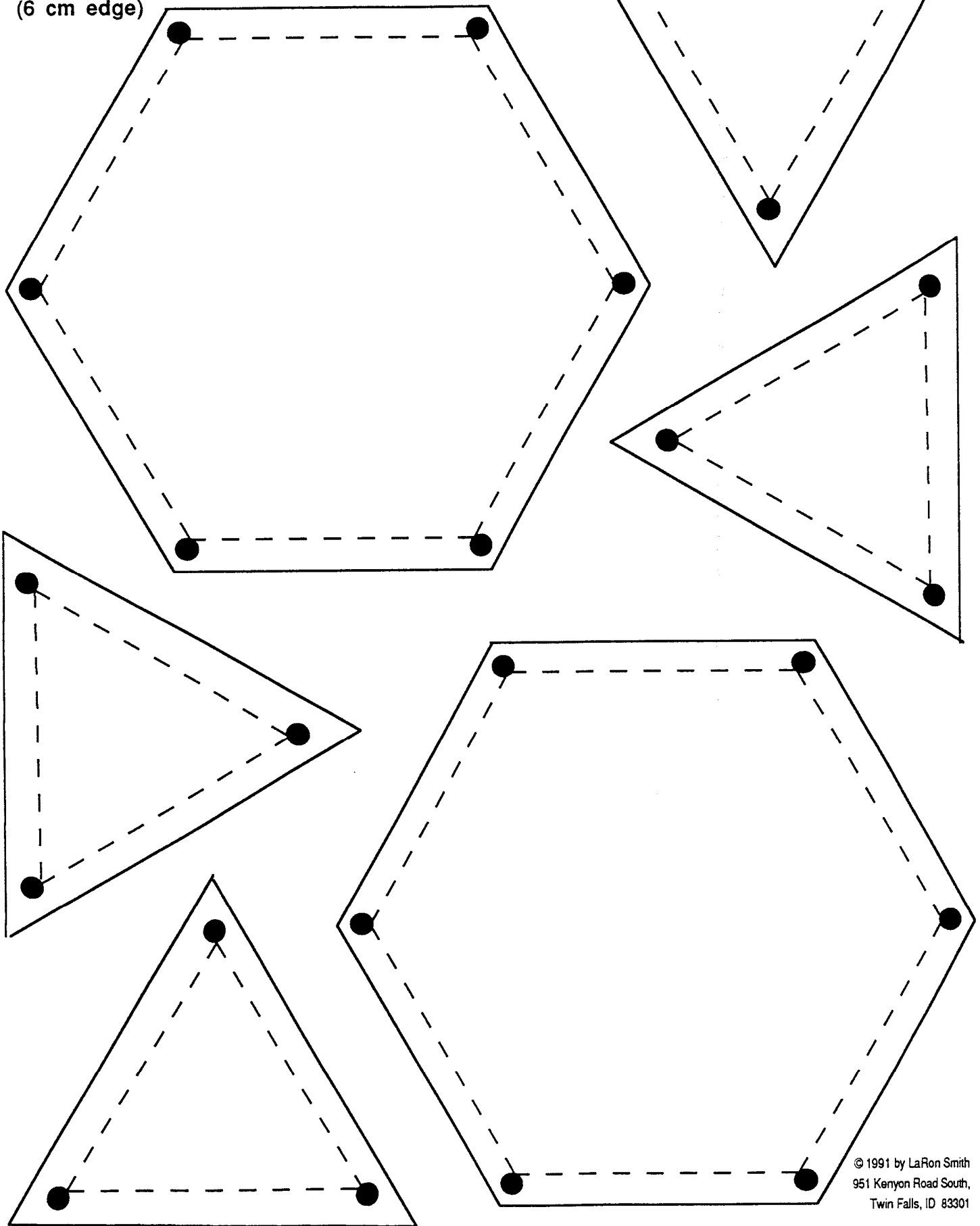


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Band-it Polyhedra

Triangles and Hexagons

(6 cm edge)



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