Think Through Math

//CODiE//
2014 SIIA CODiE WINNER
"Best Mathematics Instructional Solution"

Grades 3–Geometry

CREDIT RECOVERY PLAN

Think Through Math (TTM) is now available to utilize for Algebra 1 and Geometry credit recovery for your students. The program is simple to implement and monitor, and students will go through a rigorous set of coursework which will give them a solid foundation in Algebra 1 and will prepare them for success in Geometry and future math courses.

Program Overview

TTM's Algebra 1 credit recovery program initially enrolls students into a pathway of 60 Grade Level lessons covering 93% of the Idaho Core Algebra 1 standards. TTM's Geometry credit recovery program initially enrolls students into a pathway of 42 lessons covering 93% of the Idaho Core Geometry standards. This pathway has been constructed in conjunction with the Idaho SDE and all lessons are aligned to the Idaho Core Standards Algebra 1 and Geometry courses.

Students take a brief placement test which determines if there are additional pre-cursor lessons that need to be added into each student's pathway. Additionally, TTM monitors student progress and adds additional pre-cursors as needed to fill students' learning gaps.

Below are the Algebra 1 and Geometry Credit Recovery pathways and the associated standards to which they are aligned.

Algebra I Credit Recovery Learning Pathway

Think Through Math

Unit	*Lesson & Standards Addressed*
Expressions and Equations	Language of Algebra HSA-SSE.A.1a Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients.
Interpreting Functions	Graphs of Real-World Functions I HSF-IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Sketching Graphs of Real-World Situations HSF-IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Graphs of Real-World Functions II HSF-IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
Seeing Structure in Expressions	Evaluating Expressions with Real Numbers II HSA-SSE.A.1b Interpret expressions that represent a quantity in terms of its context. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. Simplifying Algebraic Expressions HSA-SSE.A.1b Interpret expressions that represent a quantity in terms of its context. b. Interpret complicated expressions by viewing one or more of their parts as a single entity.
Creating Equations	Writing Equations in One Variable HSA-SSE.A.1a Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. HSA-CED.A.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. HSF-BF.A.1a Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
Reasoning with Equations and Inequalities	Solving Equations with One Variable HSA-CED.A.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. Solving One-Step Equations HSA-REI.A.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. HSA-REI.B.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. Solving Multistep Equations HSA-REI.A.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. HSA-REI.B.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Algebra I Credit Recovery Learning Pathway

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Unit	*Lesson & Standards Addressed*
Creating Equations	Writing Equations with Two or More Variables HSA-SSE.A.1a Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. HSA-CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. HSF-BF.A.1a Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. Solving Literal Equations HSA-CED.A.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. HSF-BF.B.4a Find inverse functions. a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse.
The Number System	Products and Sums with Rational and Irrational Numbers* HSN-RN.B.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.
Expressions and Equations	Solving Rational and Radical Equations I* HSA-REI.A.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. Solving Rational and Radical Equations II* HSA-APR.D.7 Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. HSA-REI.A.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
Interpreting Functions	Graphing Linear Equations HSA-CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. HSA-REI.D.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). HSF-IF.C.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). HSF-BF.B.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. HSF-LE.A.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). Slope and Rate of Change HSF-IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. HSF-LE.B.5 Interpret the parameters in a linear or exponential function in terms of a context.
Building Functions	Direct Variation HSF-BF.B.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. HSF-LE.B.5 Interpret the parameters in a linear or exponential function in terms of a context.

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Unit	*Lesson & Standards Addressed*
Interpreting Functions	Equations in Slope Form HSA-REI.D.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). HSF-LE.B.5 Interpret the parameters in a linear or exponential function in terms of a context.
Creating Equations	Equations of Parallel and Perpendicular Lines HSA-CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
Reasoning with Equations and Inequalities	Inequalities HSA-CED.A.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. HSA-CED.A.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. HSA-REI.B.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. Graphing Inequalities HSA-CED.A.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. HSA-REI.D.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.
Expressions and Equations	Graphing Inequalities and Systems of Inequalities in Real-World Situations* HSA-CED.A.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. HSA-REI.D.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.
Reasoning with Equations and Inequalities	Correlation* HSS-ID.B.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. HSS-ID.B.6a Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. HSS-ID.B.6c Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. c. Fit a linear function for a scatter plot that suggests a linear association. HSS-ID.C.8 Compute (using technology) and interpret the correlation coefficient of a linear fit. HSS-ID.C.9 Distinguish between correlation and causation. Solving Systems of Equations HSA-REI.C.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. HSA-REI.C.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. HSA-REI.D.11 Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

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Unit	*Lesson & Standards Addressed*
Expressions and Equations	Factoring Expressions HSA-SSE.B.3a Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines.
Arithmetic with Polynomials and Rational Expressions	Adding and Subtracting Polynomials HSA-APR.A.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
Expressions and Equations	Simplifying Monomials HSA-SSE.A.1a Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. Using Rational Exponents to Rewrite Expressions* HSA-SSE.B.3c Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. c. Use the properties of exponents to transform expressions for exponential functions. HSN-RN.A.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. HSN-RN.A.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.
Arithmetic with Polynomials and Rational Expressions	Multiplying and Dividing Monomials HSA-APR.A.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. Multiplying Polynomials HSA-APR.A.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
Seeing Structure in Expressions	Factoring Polynomials HSA-SSE.A.2 Use the structure of an expression to identify ways to rewrite it. HSA-SSE.B.3a Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines. HSF-IF.C.8a Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
Linear, Quadratic, and Exponential Models	Graphs of Quadratic Functions HSA-CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. HSA-REI.D.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). HSF-IF.C.7a Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. HSF-IF.C.7b Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
Reasoning with Equations and Inequalities	Using the Quadratic Formula HSA-REI.B.4b Solve quadratic equations in one variable. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.

Unit	*Lesson & Standards Addressed*
Linear, Quadratic, and Exponential Models	Problem Solving with Quadratic Functions HSA-SSE.B.3a Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines. HSA-REI.B.4b Solve quadratic equations in one variable. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b. HSF-IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. HSF-BF.A.1a Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
Reasoning with Equations and Inequalities	Solving Quadratics - Completing the Square* HSA-SSE.B.3b Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. HSA-REI.B.4a Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. HSF-IF.C.7a Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. HSF-IF.C.8a Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. Complex Numbers and Complex Solutions* HSA-REI.B.4b Solve quadratic equations in one variable. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.
Linear, Quadratic, and Exponential Models	Graphs of Exponential Functions HSN-Q.A.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. HSF-IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. HSF-IF.C.8b Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. b. Use the properties of exponents to interpret expressions for exponential functions. HSF-LE.A.1a Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals; and that exponential functions grow by equal factors over equal intervals. HSF-LE.A.1b Distinguish between situations that can be modeled with linear functions and with exponential functions. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. HSF-LE.A.1c Distinguish between situations that can be modeled with linear functions and with exponential functions. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. HSF-LE.A.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

Unit	*Lesson & Standards Addressed*
Interpreting Functions	Introduction to Logarithms* HSF-IF.C.7e Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. HSF-LE.B.5 Interpret the parameters in a linear or exponential function in terms of a context. Logarithmic Functions* HSF-IF.C.7e Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. HSF-LE.B.5 Interpret the parameters in a linear or exponential function in terms of a context. Radical Functions and Their Graphs HSA-CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. HSA-REI.D.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). HSN-RN.A.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. HSN-RN.A.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents. HSF-IF.C.7b Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. Rational Functions and Their Graphs HSA-CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. HSA-REI.D.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). HSN-RN.A.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. HSN-RN.A.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents. Sequences - A Type of Function I* HSF-IF.A.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. HSF-BF.A.1a Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. HSF-BF.A.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. Sequences - A Type of Function II* HSF-IF.A.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. HSF-BF.A.1a Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. HSF-BF.A.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

Unit	*Lesson & Standards Addressed*
Building Functions	Arithmetic Sequences - Closed Form HSF-BF.A.1a Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. HSF-BF.A.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. Geometric Sequences - Closed Form HSF-IF.A.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. HSF-BF.A.1a Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. HSF-BF.A.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. Arithmetic Sequences - Recursive Form HSF-BF.A.1a Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. HSF-BF.A.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. Geometric Sequences - Recursive Form HSF-IF.A.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. HSF-BF.A.1a Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. HSF-BF.A.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
Interpreting Functions	Graphing Polynomial Functions* HSF-IF.C.7c Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. HSA-APR.B.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. Composite Functions* HSN-Q.A.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. HSF-BF.A.1b Write a function that describes a relationship between two quantities. b. Combine standard function types using arithmetic operations. HSF-BF.B.4a Find inverse functions. a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. Function Notation HSF-IF.A.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). HSF-IF.A.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Algebra I Credit Recovery Learning Pathway

Unit	*Lesson & Standards Addressed*
Statistics and Probability	Measures of Spread - Range HSS-ID.A.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). Measures of Center - Median HSS-ID.A.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). Measures of Center - Mean HSS-ID.A.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). Quartiles HSS-ID.A.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. Box Plots HSS-ID.A.1 Represent data with plots on the real number line (dot plots, histograms, and box plots). HSS-ID.A.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. HSS-ID.A.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). Sampling HSS-ID.A.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. Standard Deviation* HSN-Q.A.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
Arithmetic with Polynomials and Rational Expressions	Adding and Subtracting Rational Expressions* HSA-APR.D.7 Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. Simplifying, Multiplying, and Dividing Rational Expressions* HSA-APR.D.7 Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Unit	*Lesson & Standards Addressed*
Congruence	Defining Basic Geometric Elements* HSG-CO.A.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. Defining Transformations* HSG-CO.A.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. Rotational and Reflectional Symmetry* HSG-CO.A.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. Representing Transformations with Algebra* HSG-CO.A.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). Rigid Motion and Congruence* HSG-CO.A.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. HSG-CO.B.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. What is Proof?* HSG-CO.B.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. HSG-CO.C.9 Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints. Proving Theorems About Lines and Angles* HSG-CO.C.9 Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints. Proving Theorems About Congruent Triangles* HSG-CO.B.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. HSG-CO.C.10 Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
Similarity, Right Triangles, and Trigonometry	Problem Solving with Congruent Triangles* HSG-CO.C.10 Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point. HSG-SRT.B.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Geometry Credit Recovery Learning Pathway

Unit	*Lesson & Standards Addressed*
Congruence	Proving Theorems About Parallelograms* HSG-CO.C.11 Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals. Constructing Angles and Special Line Segments* HSG-CO.D.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line. Constructing Inscribed Figures* HSG-CO.D.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.
Modeling with Geometry	Modeling Objects with Geometric Figures* HSG-MG.A.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). Using Geometric Relationships to Solve Design Problems* HSG-MG.A.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).
Similarity, Right Triangles, and Trigonometry	Properties of Dilations I* HSG-SRT.A.1b Verify experimentally the properties of dilations given by a center and a scale factor: b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor. Properties of Dilations II* HSG-SRT.A.1a Verify experimentally the properties of dilations given by a center and a scale factor: a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. Transformations and Similarity* HSG-SRT.A.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. HSG-SRT.A.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. Problem Solving with Transformations and Similarity* HSG-SRT.B.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
Geometry	Pythagorean Theorem - Hypotenuse HSG-SRT.C.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. Pythagorean Theorem - Legs HSG-SRT.C.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. Pythagorean Theorem - Mixed Problems HSG-SRT.C.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Geometry Credit Recovery Learning Pathway

Think Through Math

Unit	*Lesson & Standards Addressed*
Similarity, Right Triangles, and Trigonometry	Proving Theorems About Similar Triangles* HSG-SRT.B.4 Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity. Similarity and Trigonometric Ratios* HSG-SRT.C.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. Problem Solving with Similarity and Trigonometric Ratios* HSG-SRT.C.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. Sine and Cosine of Complementary Angles* HSG-SRT.C.7 Explain and use the relationship between the sine and cosine of complementary angles.
Geometry	Volume of Cylinders HSG-GMD.A.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. Volume of Pyramids and Cones HSG-GMD.A.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. Volume of Spheres HSG-GMD.A.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.
Modeling with Geometry	Rates with Area and Volume* HSG-MG.A.2 Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
Geometric Measurement and Dimension	Understanding Formulas for Curved Figures* HSG-GMD.A.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments. Cross-Sections of 3-Dimensional Figures* HSG-GMD.B.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
Expressing Geometric Properties with Equations	Coordinates of Parallel and Perpendicular Lines* HSG-GPE.B.4 Use coordinates to prove simple geometric theorems algebraically. HSG-GPE.B.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). Problem Solving with Coordinates of Parallel and Perpendicular Lines* HSG-GPE.B.4 Use coordinates to prove simple geometric theorems algebraically. Dividing a Segment Proportionally* HSG-GPE.B.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio. Using Coordinates to Find Perimeters and Areas* HSG-GPE.B.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.
Circles	Chords, Radii, and Inscribed Angles* HSG-C.A.2 Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle. Radians and Area of Sectors* HSG-C.B.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Unit	Lesson & Standards Addressed
Expressing Geometric Properties with Equations	Equation of a Circle* HSG-GPE.A.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. HSG-C.A.1 Prove that all circles are similar. Problem Solving with the Equation of a Circle* HSG-GPE.A.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. HSG-GPE.B.4 Use coordinates to prove simple geometric theorems algebraically.
Circles	Quadrilaterals Inscribed in Circles* HSG-C.A.3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
Similarity, Right Triangles, and Trigonometry	Law of Sines and Law of Cosines* HSG-SRT.D.11 Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
Conditional Probability and the Rules of Probability	Using Area Models for Compound Probability* HSS-CP.A.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. HSS-CP.B.9 Use permutations and combinations to compute probabilities of compound events and solve problems.

Lesson Overview

Each lesson follows a Gradual Release model. This means that the program starts by modeling how to do the math and gradually releases responsibility to the student. The beginning activities have more support and gradually remove support until the Post Quiz, where the student must do the math completely on their own. The Problem Solving activity serves as a part of the instructional component within TTM. Two of the activities, Guided Learning and the Problem Solving Process, provide access to a Live Teacher.

Below is a summary of the activities within each lesson. Not every lesson has all of these activities.

- **PRE-QUIZ:** If the student thinks (s)he knows the lesson material, the student may opt to take the Pre-Quiz. If the student scores 80% or better on the Pre-Quiz, he or she skips the lesson, and the lesson path adapts to match student need.
- **WARM UP:** In this activity, the student sees items that are precursors to the standard for this lesson. This activates students' prior knowledge to increase success. It reinforces the idea that each piece of math learning is connected to other math understandings.
- **GUIDED LEARNING:** Guided Learning is a series of items that span the key parts of the standard of the lesson. This is a very interactive type of learning activity where the student is posed a question and may choose to try to answer right away, or get more help before they answer. Once a student exhausts all of the Math Coach help, he or she can get help from a Live Teacher.

- **PROBLEM SOLVING PROCESS:** The Problem Solving Process activity focuses on a 5-step process for solving problems that students can apply to any problem that can be solved by writing an equation. This process bolsters students' confidence that they can get started solving any problem without anxiety. Access to Live Teachers is also available here.

  During this activity, students practice the problem-solving process:

  - Analyzing given information
  - Formulating a plan or strategy
  - Determining a solution
  - Justifying the solution
  - Evaluating the problem-solving process and the reasonableness of the solution

- **INDEPENDENT PRACTICE:** In the Independent Practice activity, students get plenty of practice with more problems
  on the target standard. They work more independently in this activity. The student has correction on incorrect answer, but the instructional support from the coach and live teacher is no longer available.
- **POST-QUIZ:** At the end of each lesson, students prove whether they know the mathematics totally independently (the final step in the gradual release process). If they get 60% or better on this activity, they pass the lesson.

Monitoring Student Progress

We have created a very easy to implement and monitor program for credit recovery.

As students work through their pathway, reports on student progress are available through the teacher portal. These are easy to use reports that indicate how much work students are doing as well as how much progress they are making toward completing the Target and pre-cursor lessons on their pathway.

In order to insure that students demonstrate proficiency on the content, we highly recommend that you require students to maintain a math journal to document their work and serve as a portfolio of their progress. Teachers are free to implement this in any way that they see fit. However, TTM has a Math Journal, along with suggestions for its use, which is available under the support tab of the teacher page.

Grading

The grading process for this program is very simple. Students should be given credit for Algebra 1 if they:

- Pass at least 70% of their Grade Level lessons (the lessons listed on this document)
- Submit a math journal that meets the expectations set by the monitoring teacher
  - At minimum, this journal should include student work for all applicable problems the student completed while working in TTM
  - Strategies for using a math journal are provided below.

Using a Math Journal in the Think Through Math Classroom

Why journaling?: The NCTM's Curriculum and Evaluation Standards states "The assessment of students' ability to communicate mathematics should provide evidence that they can express mathematical ideas by speaking, writing, demonstrating and depicting them visually." (1989, 214)

Having students use the TTM Math Journal along with the computer is very important to the success of the TTM program in your classroom.

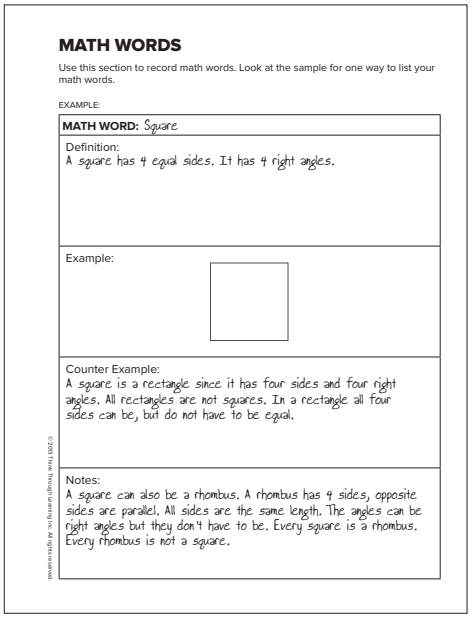
TTM Journals:

- Provide an accountability system to ensure students are working carefully and thoughtfully
- Prepare students for computer-based state testing
- Create a record/portfolio of students' progress through TTM
- Can address many of the habits of mind and/or mathematical standards outlined in new state standards

Goal setting: It is important that students set weekly goals. These goals help keep students focused and allow them an opportunity to track their own progress.

Week #	Goal for the Week	Goal Met?
Week 1	Customize my avatar and finish the Placement Test	Yes
Week 2	Complete 2 Lessons	No
Week 3	Finish 2 lessons and spend 30 minutes outside of school	Yes

Have the students use a chart similar to this one to track their progress on TTM each week. By using small weekly goals, students will stay on track and make continual progress.



MATH WORDS

Use this section to record math words. Look at the sample for one way to list your math words.

EXAMPLE:

MATH WORD: Square

Definition:
A square has 4 equal sides. It has 4 right angles.

Example:

Counter Example:
A square is a rectangle since it has four sides and four right angles. All rectangles are not squares. In a rectangle all four sides can be, but do not have to be equal.

Notes:
A square can also be a rhombus. A rhombus has 4 sides, opposite sides are parallel. All sides are the same length. The angles can be right angles but they don't have to be. Every square is a rhombus. Every rhombus is not a square.

Math vocabulary building: Vocabulary is critical to mathematical success in the classroom:

- Comprehension improves when you know what the words mean.
- Math is a language many do not speak fluently.
- When children and adolescents improve their vocabulary, their academic and social confidence and competence improve, too

Math Journal Strategies

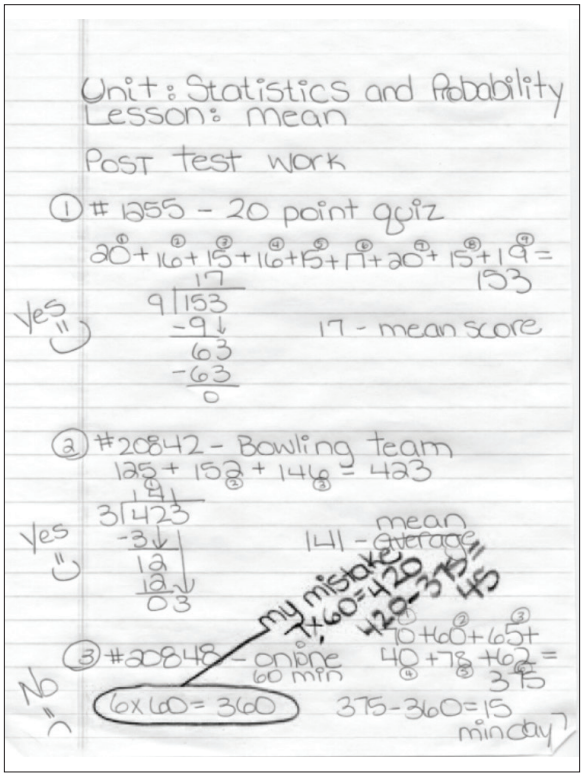
Level 1: Show work on Pre-Quiz and Post-Quiz problems. Since the Pre and Post Quizzes are the assessment components and also the only areas in the program without any help or feedback, it's one of the most useful/important places to make sure students are carefully working through each problem.

Advantages

- Simple to implement – great way to start using a journal for the first time
- Keeps students accountable for their work on the quizzes
- Allows teachers to easily see where they are struggling conceptually (Teachers can have students bring up the journal if they see student is struggling in a given area)
- Simple to track (Teachers can collect journals periodically and spot-check for work)
- Doesn't require a lot of extra time

Challenges

- Doesn't provide any insight into other activities
- Doesn't provide a deeper insight into why they are missing the problems
- Doesn't address the "communicating about math" components embedded in the new standards



JOURNALING STRATEGIES

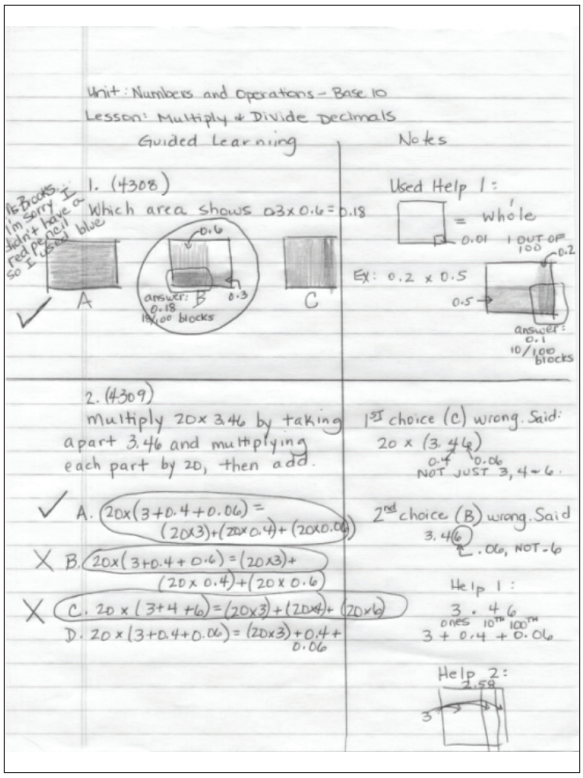
Level 2: Add on showing work in Guided Learning activity. Also require students to ask for help in Guided Learning and take down notes for the help provided.

Advantages

- Provides more insight into the conceptual building blocks of the concepts that students work through in the Guided Learning activity
- Encourages students to take better advantage of the help provided in Guided Learning
- Gives students notes from which to refer while working on additional material
- Gives teachers notes to refer to while working with in small group or individual settings

Challenges

- Same as Level 1
- Takes more time for students to complete a lesson



JOURNALING STRATEGIES

Level 3: For a subset of the problems in the Practice, Pre-Quiz and Post-Quiz activities, have students explain what they did to solve the problem.

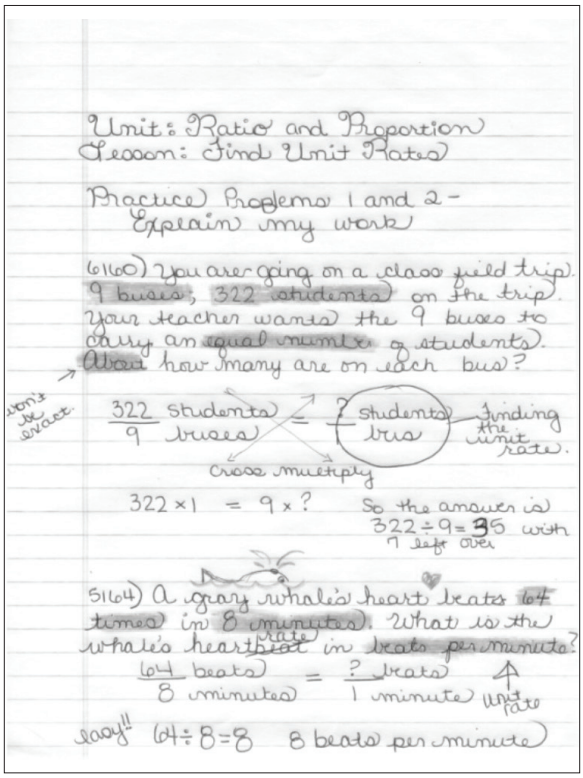
Advantages

- Students begin to work on being able to communicate their understanding of the concept behind each problem (mathematical practice standards)
- Students get to select the problems that they understand the best.
- Gives teacher much more insight into level of conceptual understanding and/or misconceptions

Challenges

- Initially difficult for students to communicate clearly about mathematics
- Good idea for teacher to provide exemplar models for students to follow
- Students can also follow examples provided when they get problems correct in Guided Learning activity
- Takes more time for students to complete a lessons

Variation: Have students explain where they got stuck on a problem that they got wrong.



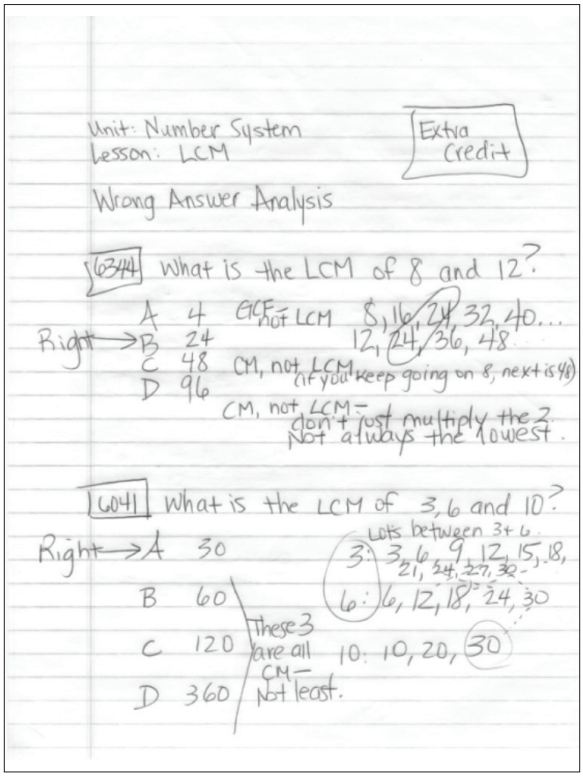
Level 4: For a subset of multiple choice problems (or possibly for extra credit), have students perform wrong answer analysis – explain what is the misconception behind the wrong answers in multiple choice problems). NOTE: Not all wrong answers model a specific misconception.

Advantages

- Allows students to demonstrate deeper level of understanding by identifying incorrect strategies that are commonly used

Challenges

- Initially very difficult for students to do. We highly recommend some modeling from teachers first
- Takes more time to finish a lesson



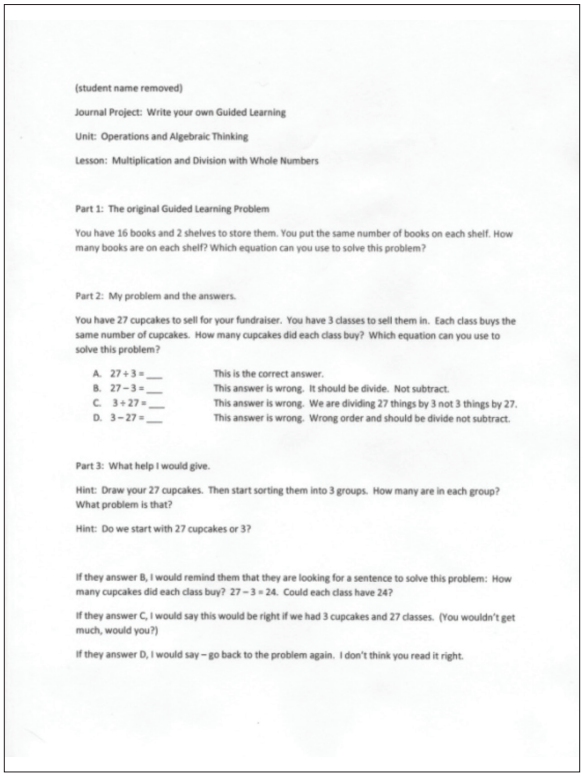
Level 5: Have students construct a problem similar to one of the real-world problems they encounter – preferably in the Guided Learning activity. Problem should include a real-world scenario, distractors and feedback that they would provide to help students understand the concept and solve the problem successfully.

Advantages

- Very engaging activity with students
- Allows students to be creative
- Forces students to stretch the depth of their understanding of concepts as well as the common pitfalls that many students encounter
- Constructing feedback forces students to think about how to teach the concept to other students

Challenges

- Larger project – needs time, clear outline of expectations, rubric for grading
- Time consuming – would only do once or twice a semester at most



(student name removed)

Journal Project: Write your own Guided Learning

Unit: Operations and Algebraic Thinking

Lesson: Multiplication and Division with Whole Numbers

Part 1: The original Guided Learning Problem

You have 16 books and 2 shelves to store them. You put the same number of books on each shelf. How many books are on each shelf? Which equation can you use to solve this problem?

Part 2: My problem and the answers.

You have 27 cupcakes to sell for your fundraiser. You have 3 classes to sell them in. Each class buys the same number of cupcakes. How many cupcakes did each class buy? Which equation can you use to solve this problem?

A.	27 ÷ 3 = ___	This is the correct answer.
B.	27 – 3 = ___	This answer is wrong. It should be divide. Not subtract.
C.	3 + 27 = ___	This answer is wrong. We are dividing 27 things by 3 not 3 things by 27.
D.	3 – 27 = ___	This answer is wrong. Wrong order and should be divide not subtract.

Part 3: What help I would give.

Hint: Draw your 27 cupcakes. Then start sorting them into 3 groups. How many are in each group? What problem is that?

Hint: Do we start with 27 cupcakes or 3?

If they answer B, I would remind them that they are looking for a sentence to solve this problem: How many cupcakes did each class buy? 27 – 3 = 24. Could each class have 24?

If they answer C, I would say this would be right if we had 3 cupcakes and 27 classes. (You wouldn't get much, would you?)

If they answer D, I would say – go back to the problem again. I don't think you read it right.