Claim 1: Concepts and Procedures Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.	
Content Domain: **The Number System**	
Target A [s]: Know that there are numbers that are not rational, and approximate them by rational numbers. (DOK Levels 1, 2) Tasks will ask students to approximate irrational numbers on a number line or as rational numbers with a certain degree of precision. This target may be combined with 8.EE Target B (e.g., by asking students to express the solution to a cube root equation as a point on the number line).	
Standards: 8.NS.A, 8.NS.A.1, 8.NS.A.2	**8.NS.A Know that there are numbers that are not rational, and approximate them by rational numbers.** **8.NS.A.1** Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion, which repeats eventually into a rational number. **8.NS.A.2** Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of* $\sqrt{2}$ *show that* $\sqrt{2}$ *is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*
Related Below-Grade and Above-Grade Standards for Purposes of Planning for Vertical Scaling: 7.NS.A, 7.NS.A.2, 7.NS.A.3 N-RN.A, N-RN.A.1, N-RN.A.2,N-RN.B, N-RN.B.3	**Related Grade 7 Standards** **7.NS.A Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.** **7.NS.A.2** Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts. c. Apply properties of operations as strategies to multiply and divide rational numbers. d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats. **7.NS.A.3** Solve real-world and mathematical problems involving the four operations with rational numbers.

	Related High School Standards **N-RN.A Extend the properties of exponents to rational exponents.** **N-RN.A.1** Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.* **N-RN.A.2** Rewrite expressions involving radicals and rational exponents using the properties of exponents. **N-RN.B Use the properties of rational and irrational numbers.** **N-RN.B.3** Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.
DOK Levels:	1, 2
Achievement Level Descriptors:	
RANGE Achievement Level Descriptor (Range ALD) Target A: Know that there are numbers that are not rational and approximate them by rational numbers.	**Level 1** Students should be able to identify square roots of numbers less than 100; identify pi as not rational; and understand that every rational number has a decimal expansion.
	Level 2 Students should be able to identify approximate locations of familiar irrational numbers on a number line; identify numbers as rational or irrational; and convert between fractions and terminating decimals.
	Level 3 Students should be able to use rational approximations of irrational numbers to locate them on a number line and to make numerical comparisons; convert between fractions and repeating decimals; and compare rational numbers.
	Level 4 Students should be able to approximate irrational numbers to a specified level of precision and should be able to use the approximations to solve problems or estimate the value of an expression.
Evidence Required:	1. The student classifies real numbers as rational or irrational. 2. The student converts a repeating decimal into a fraction. 3. The student writes approximations of irrational numbers as rational numbers. 4. The student compares the sizes of irrational numbers by using rational approximations of irrational numbers. 5. The student approximates the locations of irrational numbers on the number line by using rational approximations of irrational numbers.
Allowable Response Types:	Multiple Choice, multiple correct response; Multiple Choice, single correct response; Matching Tables; Equation/Numeric; Drag and Drop; Graphing
Allowable Stimulus	rational numbers, irrational numbers, expressions involving irrational

Materials:	numbers, explanations of processes, number lines (showing tenths or hundredths), square roots, cube roots, pi, repeating bar, repeating and terminating decimals
Construct-Relevant Vocabulary:	rational number, irrational number, repeating decimal, terminating decimal, square root, pi (π)
Allowable Tools:	None
Target-Specific Attributes:	Irrational numbers should be square roots, cube roots, or pi (π). Calculators are not allowed for this target.
Non-Targeted Constructs:	
Accessibility Guidance:	Item writers should consider the following Language and Visual Element/Design guidelines[1] when developing items. Language Key Considerations: • Use simple, clear, and easy-to-understand language needed to assess the construct or aid in the understanding of the context • Avoid sentences with multiple clauses • Use vocabulary that is at or below grade level • Avoid ambiguous or obscure words, idioms, jargon, unusual names and references Visual Elements/Design Key Considerations: • Include visual elements only if the graphic is needed to assess the construct or it aids in the understanding of the context • Use the simplest graphic possible with the greatest degree of contrast, and include clear, concise labels where necessary • Avoid crowding of details and graphics Items are selected for a student's test according to the blueprint, which selects items based on Claims and targets, not task models. As such, careful consideration is given to making sure fully accessible items are available to cover the content of every Claim and target, even if some item formats are not fully accessible using current technology.[2]
Development Note:	An item measuring the "explain" part of this target and standard may be assessed in Claim 3.

[1] For more information, refer to the General Accessibility Guidelines at: http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/05/TaskItemSpecifications/Guidelines/AccessibilityandAccommodations/GeneralAccessibilityGuidelines.pdf

[2] For more information about student accessibility resources and policies, refer to http://www.smarterbalanced.org/wordpress/wp-content/uploads/2014/08/SmarterBalanced_Guidelines.pdf

Task Model 1

Response Type: Matching Tables

DOK Level 1

8.NS.A.1
Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion, which repeats eventually into a rational number.

Evidence Required:
1. The student classifies real numbers as rational or irrational.

Tools: None

Prompt Features: The student classifies numbers as rational or irrational.

Stimulus Guidelines: Item difficulty can be adjusted via these methods:

- Rational numbers are positive; irrational numbers are pi or $\sqrt{2}$.
- Rational numbers can be positive or negative; irrational numbers are in the form $\sqrt{x}$, where x is less than 90.
- Rational numbers can be positive or negative; they can be expressed as mixed numbers, fractions, decimals, or repeating decimals; irrational numbers include numerical expressions with pi and radicals.

TM1
Stimulus: The student is presented with a table of four to five rational and irrational numbers.

Example Stem: Determine for each number whether it is a rational or irrational number.

Number	Rational	Irrational
$\frac{4}{7}$		
$\sqrt{30}$		
$\frac{21}{\sqrt{4}}$		
π		
-27		

Rubric: (1 point) The student correctly classifies each number (e.g., irrational numbers are $\sqrt{30}$ and π, all others are rational).

Response Type: Matching Tables

Task Model 2 **Response Type: Equation / Numeric** **DOK Level 1** **8.NS.A.1** Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion, which repeats eventually into a rational number. **Evidence Required:** 2. The student converts repeating decimals to fractions. **Tools:** None	**Prompt Features:** The student converts a repeating decimal to a fraction. **Stimulus Guidelines:** Item difficulty can be adjusted via these methods: • Decimals with one repeating digit in the tenths place. • Decimals with multiple repeating digits starting in the tenths place (e.g.,$0.\overline{24}$, $8.\overline{125}$), the hundredths place, or the thousandths place (e.g., $0.0\overline{42}$, $3.07\overline{6}$). **TM2** **Stimulus:** The student is presented with a decimal with a repeating bar over the last digit(s). **Example Stem:** Enter a fraction equivalent to $0.\overline{2}$. Use only whole numbers for numerators and denominators. **Rubric:** (1 point) Student enters an equivalent fraction to the repeating decimal (e.g., 2/9). **Response Type:** Equation/Numeric

Task Model 3 **Response Type: Multiple Choice, single correct response** **DOK Level 1** **8.NS.A.2** Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). **Evidence Required:** 3. The student writes approximations of irrational numbers as rational numbers. **Tools:** None	**Prompt Features:** The student approximates the value of an irrational number to the nearest whole number. **Stimulus Guidelines:** • Irrational numbers should involve square roots, cube roots, or pi (π). • The expression could include addition, subtraction, multiplication, and division operations. • Item difficulty can be adjusted via these methods: o An irrational number between $\sqrt{2}$ and $\sqrt{100}$. o An irrational number or expression and the degree of precision is to the whole number. o An irrational expression and the degree of precision is to the tenths. **TM3a** **Stimulus:** The student is presented with an irrational number. **Example Stem:** Which number is the closest approximation to $\sqrt{167}$? A. 12 B. 13 C. 83 D. 84 **Answer Choices:** Distractors include incorrect rounding up or down and misinterpreting the square root sign as “divide by 2”. **Rubric:** (1 point) The student correctly identifies the closest approximation (e.g., B). **Response Type:** Multiple Choice, single correct response

Task Model 3 **Response Type: Equation / Numeric** **DOK Level 2** **8.NS.A.2** Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). **Evidence Required:** 3. The student writes approximations of irrational numbers as rational numbers. **Tools:** None	**Prompt Features:** The student provides a decimal approximation of a numerical expression whose value is irrational. **Stimulus Guidelines:** • Irrational numbers should involve square roots or pi (π). • The expression could include the operations of addition, subtraction, multiplication, and division. • Item difficulty can be adjusted via these methods: o The numerical expression is a single irrational number between $\sqrt{2}$ and $\sqrt{100}$ (including pi). o The numerical expression is a single number or a more complex expression and the approximation is to the nearest whole number. o The numerical expression is more complex than a single number and the approximation is to the nearest tenth. **TM3b** **Stimulus:** The student is presented with an expression whose value is irrational. **Example Stem:** Enter the approximate value of $2\sqrt{47}$ to the nearest tenth. **Rubric:** (1 point) Student gives the correct approximation at the specified degree of approximation (e.g., 13.6 or 13.7). **Response Type:** Equation/Numeric

Task Model 3 **Response Type: Multiple Choice, single correct response** **DOK Level 2** **8.NS.A.2** Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). **Evidence Required:** 3. The student writes approximations of irrational numbers as rational numbers. **Tools:** None	**Prompt Features:** The student identifies the range in which the value of a numerical expression whose value is irrational falls. **Stimulus Guidelines:** • Irrational numbers should involve square roots, cube roots, or pi (π). • The expression could include the operations of addition, subtraction, multiplication, and division. • Item difficulty can be adjusted via these methods: o The numerical expression is a single irrational number between $\sqrt{2}$ and $\sqrt{100}$ (including pi). o The numerical expression is a single number or a more complex expression and the approximation is to the nearest whole number. o The numerical expression is more complex than a single number and the approximation is to the nearest tenth **TM3c** **Stimulus:** The student is presented with a numerical expression whose value is irrational. **Example Stem:** Which range contains the value of $\sqrt{(16 + 9 + 20)}$? A. between 6.6 and 6.8 B. between 7.5 and 7.7 C. between 16.8 and 17.0 D. between 22.4 and 22.6 **Answer Choices:** Ranges may be whole numbers or decimals to the tenths. The distractors are ranges that are produced by errors in order of operations, errors in rounding, trying to remove perfect squares from the addends, or interpreting the square root as "divide by 2." **Rubric:** (1 point) Student selects the correct range (e.g., A). **Response Type:** Multiple Choice, single correct response

Task Model 4 **Response Type: Multiple Choice, multiple correct response** **DOK Level 2** **8.NS.A.2** Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). **Evidence Required:** 4. The student compares the sizes of irrational numbers by using rational approximations of irrational numbers. **Tools:** None	**Prompt Features:** The student compares the sizes of irrational numbers. **Stimulus Guidelines:** • Four or five expressions are to be given as options. • At least two of the options must be correct responses. • Item difficulty can be adjusted by varying the complexity of the stem and answer choices: o They are simple numerical expressions with rational or irrational values such as integers, fractions, decimals to the tenths, radicals up to $\sqrt{30}$, integers with a fractional exponent, or multiples of pi (π). o They are more complex expressions with rational or irrational values involving one operation (including exponentiation) with integers, fractions, decimals to the hundredths place, radicals up to $\sqrt{120}$, and multiples of pi. o They are more complex expressions with rational or irrational values involving one or more operations (including exponentiation) with integers, fractions, decimals, radicals, exponents, multiples of pi. **TM4a** **Stimulus:** The student is presented with a list of numerical expressions some of which have irrational values. **Example Stem:** Select **all** expressions that have a value greater than 5. A. 2π B. $\frac{10}{\sqrt{3}}$ C. $3 + \sqrt{2}$ D. $5.7 - \frac{6}{\sqrt{20}}$ **Rubric:** (1 point) Student selects all the appropriate expressions (e.g., A and B). **Response Type:** Multiple Choice, multiple correct response

Task Model 4

Response Type: Matching Tables

DOK Level 1

8.NS.A.2
Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2).

Evidence Required:
4. The student compares the sizes of irrational numbers by using rational approximations of irrational numbers.

Tools: None

Prompt Features: The student selects true statements about the comparison of two numerical expressions, one or both of which has an irrational value.

Stimulus Guidelines: Item difficulty can be adjusted via these methods:

- Expressions being compared may be single numbers written as fractions, decimals to the tenths, radicals up to $\sqrt{30}$, integers with a fractional exponent, or multiples of pi (π).
- Expressions being compared may be more complex, involving one operation with fractions, decimals to the tenths, radicals up to $\sqrt{120}$, integers with a fractional exponent, or multiples of pi (π).

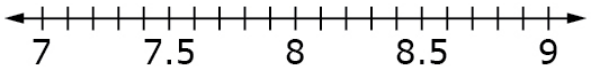
TM4b
Stimulus: The student is presented with an inequality involving an irrational number.

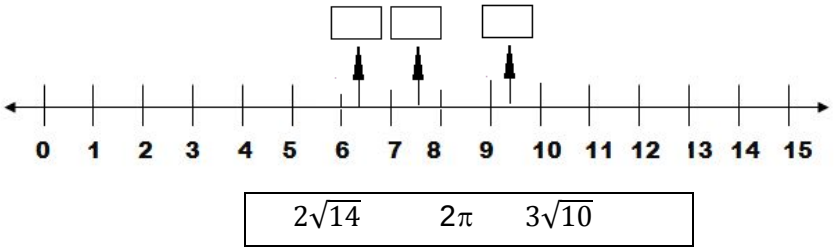
Example Stem: Select True or False to indicate whether each comparison is true.

	True	False
$\frac{4}{7} > \sqrt{19}$		
$\sqrt{40} > 7$		
$\frac{20}{\sqrt{30}} > \frac{2}{3}$		

Rubric: (1 point) Student selects True or False correctly (e.g., F, F, T). In each inequality, one of the sides must contain an irrational number.

Response Type: Matching Tables

Task Model 5 **Response Type: Graphing** **DOK Level 1** **8.NS.A.2** Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). **Evidence Required:** 5. The student approximates the locations of irrational numbers on the number line by using rational approximations of irrational numbers. **Tools:** None **Accessibility Note:** Graphing items are not currently able to be Brailled. Minimize the number of items developed to this TM. **Version 3 Update:** Retired TM5a	**Prompt Features:** The student is prompted to plot the approximate value of an irrational number onto a number line. **Stimulus Guidelines:** • Irrational numbers should involve square roots or pi (π). • Item difficulty can be adjusted via these methods: o approximation is to the nearest whole number o approximation is to the nearest tenth. **TM5b** **Stimulus:** The student is presented with a number line and an irrational number. **Example Stem:** Use the Add Point tool to approximate the value of $\sqrt{78}$ to the nearest tenth on the number line.  **Interaction:** Student will use the Add Point tool to graph a point on a number line containing snap-to regions at every tic mark. Add Point and Delete tools should be provided. **Rubric:** (1 point) Student plots a point at the correct approximation (e.g., 8.8). **Response Type:** Graphing

Task Model 5 **Response Type: Drag and Drop** **DOK Level 1** **8.NS.A.2** Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). **Evidence Required:** 5. The student approximates the locations of irrational numbers on the number line by using rational approximations of irrational numbers. **Tools:** None **Accessibility Note:** Drag and Drop items are not currently able to be Brailled. Minimize the number of items developed to this TM.	**Prompt Features:** The student is prompted to drag numerical expressions whose value is irrational onto a number line to show the approximate locations of the values. **Stimulus Guidelines:** • Irrational numbers should involve square roots, cube roots, or π. • The expression could involve any of the four operations. • Items difficulty can be adjusted via these methods: o Expressions are π or in the form of $\sqrt{x}$ where x is a positive integer less than 30. o Expressions involve addition, subtraction, or multiplication of π or $\sqrt{x}$ where x is a positive integer less than 30. o Expressions involve division or more than one operation, including taking radicals. **TM5c** **Stimulus:** The student is presented with a number line and three expressions containing irrational numbers. **Example Stem:** Drag each expression to the number line to show the approximate value.  $2\sqrt{14}$ 2π $3\sqrt{10}$ **Interaction:** A palette on the bottom should be given for the three one-time use irrational expressions. Students should drag the expressions into the appropriate boxes on the number line. There should be an arrow from the number line to the box indicating exactly at which tic mark the expression should be placed. **Rubric:** (1 point) Student places all three expressions in the correct location (e.g., 2π, $2\sqrt{14}$, $3\sqrt{10}$). **Response Type:** Drag and Drop