Idaho State Department of Education Negotiated Rulemaking

Public Comment Sign in Sheet

IDAHO CONTENT STANDARDS: MATHEMATICS

April 21, 2016

Idaho Falls, Idaho

Full Name	Signature	City	Stakeholder Group (circle one or more)
Suketu Gandhi	Suketu Gandhi	Idaho Falls	Admin Counselor (Parent) Student Teacher Other
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Please print clearly.

Idaho State Department of Education Negotiated Rulemaking

Public Comment Sign in Sheet

Mathematics

DATE: April 22, 2016 **LOCATION:** Pocatello

Full Name	Signature	City	Stakeholder Group (circle one or more)
Rose Larson	Rose Larson	Blackfoot	(Admin) Counselor Parent Student Teacher Other
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Please print clearly.

Date: April 21, 2016
Name: **Suketu Gandhi**

Signature: Suketu Gandhi

City: **Idaho Falls**
State: **Idaho**
Stakeholder Group: **Parent**

Reference: Common Core Mathematics Standards Revision

[redacted]

Suketu R. Gandhi
Parent, Idaho Falls, School District 91

Common Core Mathematics Standard Comments
April 21, 2016

Grade	Domain	Comments on Cluster/Standard
KG	K.CC	Students should be able to write numbers upto 50. They should know how to read numbers upto 100.
KG	K.MD	Students should know measurements in addition to length, and weight: Time, and money.
1	1.0A	Students should learn how to add/subtract numbers by carrying and borrowing.
1	1.MD	Students should also know the calendars. They should know how to write thousands, as it is related to identification of year. Money should also be included here. There is a direct relationship between money and expression of fraction in geometry.
2	Overview	Students should learn how to do multiplications as an extension of repeated addition. Since the numbers are restricted to 100, they should master multiplication tables from ×1 to ×10.
2	2.0A.2, 2.NBT.8	Students must be fluent with addition and subtraction with carrying and borrowing by this time. Students can do mental addition with 'odd-ball' numbers (e.g., 38 + 45) by adding the 10's digit first, and adding the one's digit. It involves adding a single digit at a time (i.e., 30+40 + 8+5 in the previous example. Students should be able to add 3-4 lines of multi-digit numbers, and be in position to carryout subtractions of multi-digits number.
2	2.MD	Missing quantity in the measurements is weight. Students should know the meaning of seconds on time scale. If students are to learn 'analog' (or hand dial) clocks (watch), they should know multiplication tables.
2	General comments	What about expression of fractions? It is important for students to know how to express fraction, and do simple addition and subtraction with pictures in a same the way as they done previously in the 1st grade with whole numbers. This will help students learn measurements on fraction of an inch, or fraction of a meter/centimeter, ½ hour, etc. Students should be taught addition of sequences of numbers via Carl Friedrich Gauss approach. They know multiplication of numbers, and pattern.
3	3.0A.7	Multiplication of two single digit number is bit too late. It should have been taught in 2nd grade. Division of a number by a single digit has to be taught in 3rd grade. In 3rd grade, multiplication of a two or higher digit number by 2- or 3- or 4-digit number should be taught. Know times tables upto ×20.
3	General Comment	Concepts of positive and negative numbers should be introduced in 3rd grade. Addition and subtraction is the topic that should be taught here. It is appropriate to discuss this topic as students have become fluent with addition and subtraction. This topic should not take long time to reach. It can be easily taught by giving analogies with sky scraper having basement,

Add Roman numerals but not for BAT-2 math.

3	General Comment	or how deep is water (e.g., in swimming pool, rivers, lakes or in the sea). This is related to measurement of length. This topic is important for science when students learn about electricity, atoms. There is no topic on pattern listed. Students should be taught addition of sequences of numbers via Carl Friedrich Gauss approach. They know multiplication of numbers, and pattern. This integrates their knowledge on addition with multiplications.
4	General comment	Students should know mental math tricks for multiplication. For example, there are ways to teach how to multiply a number ×4, ×5, ×11, ×25, ×111. There are quick ways to teach students how to square a number (e.g., 65×65, 28×28), etc. All of these concepts are ready to be taught in 4th grade as the students have the maturity. This will be utilized in fractions, and geometry (i.e., Pythagoras theorem). There is no topic on volume. Volume is very much of relevance. One can relate surface area of a box (i.e., related to how much gift wrapping paper needed) with the volume (i.e., how much water it can hold). Students should know something about speed/velocity in this grade. Students know about speed from automobiles & riding bicycles or skateboard or scooters. Students should learn how to multiply and divide by any combination of positive and negative numbers. Topic on positive and negative numbers are important to learn about electricity.
4	4.0.A.4	Introduction of prime numbers is too early in for this grade. Let the students have practice in factoring of a large whole numbers without learning about prime numbers. This is essential for doing arithmetic operation with fractions. Prime number is appropriate topic in 6th grade. It will be faster to teach this topic in 6th grade when students have the necessary practice and maturity in factoring of numbers. Students must master division of a number by two or three digit number. The examples of the pattern provided are at 2nd grade level. Instead, a pattern should be commensurate with the education. Proper example is a way to simplify answers on addition of sequences of number (i.e., odd numbers, or even numbers, etc.) and should be able to relate this via Gauss method of addition.
4	4.0A.5	There should be explicit mention of operation of pattern. Here, students should be exposed to adding sequence of numbers, which is integrates addition & multiplication. It exposes students to understand the concept of average without using the word *average*.
4	4.NBT.4	Students should have matured on addition and subtraction of multidigit numbers by end of 2nd grade. They should utilize addition to solve word problems (or other multi-step problems).

4	4.NBT.5	Students should have mastered how to multiply multi-digit number by 2 or 3 digit number (i.e., 345×27). This topic is appropriate for 3rd grade.
4	4.NBT.6	Students should divide a multi-digit number by two & three digit numbers (e.g., 4237 ÷ 19) and express remainder. Division of a multi-digit number by a single digit number is suitable in 3rd grade.
4	4.G.3	In addition, symmetry can be identified due to rotations. Symmetry should not be restricted to mirror image. Multiplication of a number by illustration of area is also suitable in 3rd grade when students learn more about the area of square and rectangle shapes.
4	4.NF.5	This topic should taught in conjunction with money as in dollars & cents.
5	Overview	Various topics in multiplications of multidigit number by two or more digit number & division of multidigit number by two or more digit number should have been completed in 4th grade. Doing this topic here wastes valuable time, and prevents going into depth for fractions and geometry, and utilization of math tricks to compute area and volume.
5	5.NF.2, 5.NF.6	Glad to see a visual (or graphical approach) to teach fraction. This is extremely important.
5	5.G.1	This topic presupposes that students know about positive & negative numbers. However, in 3rd & 4th grade students have not learned this topic. Thus, students should be exposed to addition & subtraction of positive and negative number before teaching them about the coordinate system.
5	5.G.2	Students should not be restricted to the first quadrant (or quadrant I) for graphing purpose. Students will deal with situations in real life such as left/right, up/down or east/west and north/south when reading a map. Thus, it is appropriate that students learn coordinate system in all four quadrants with positive & negative numbers.
5	General Comments	Important pieces are missing in the studies of fraction: Area of a triangle. This is integration of fraction and geometry. Geometry tells us that an area of triangle is ½ of rectangle. One has to form rectangles from different shapes of triangle. The decimal formation of fraction is proper thing to do, but no mention is made that multiplication & division of decimals. It is noted that at times conversion of decimal into fraction can be easily solved easily. This is important for word problem when students must identify strategies. For example, to solve problem 0.375×0.6, it is better to convert to fraction: 3/8×6/10 = 9/40, and one can convert to decimal easily (9/40 = 10/40 - 1/40 = 0.25-0.025 = 0.0225). Symmetry topic in geometry is missing. Furthermore, with the knowledge on the area of triangle, squares, fraction one can derive the Pythagoras theorem. This topic integrates quick mental mathematics to find a square of a number, a topic that is suitable for 3rd grade students.

5	General Comments	There is no mention of the topic on percent (%). This is directly related to fraction. Ratios are very much related to the fraction. It is easy to teach this topic as one can find ratio of length and determine the ratio of area of similar shapes. Mental mathematics for finding square of a number will be important here.
6	Overview	Some of the goals in grade 6 should have been done earlier. Examples are the standard arithmetic operation involving positive and negative numbers (3^{rd} and 4^{th} grade), area of triangles (5^{th} grade), and ratio (5^{th} grade). Algebraic expression is natural extension if one uses graphical (visual) approach to solve complex problems.
6	6.RP.1-6.RP.3	This topic should have been taught in 5^{th} grade. There are topics related to percentage, length and area from previous grades.
6	6.NS.2	This topic should have been completed in 5^{th} grade, as it relates to fraction.
6	6.NS.5	This topic should have been introduced in 3^{rd} grade (for the addition & subtraction), and 4^{th} grade (for the multiplications & division). Addition and subtraction are adequately handled by moving on a number line in either the positive or the negative direction. For the cases of multiplication & division, connection has to be made in mathematical terms: Multiplication of a *positive* and a *negative* number is negative number since it is a *repeated addition*. Similarly, division of a positive and negative number is a negative number as it is *regrouping*. In the case of multiplication & division of a *negative* number by a *negative* number, one should relate the outcome through the definition inverse & identity operation. This approach to teaching reinforces the concept of identity and inverse operations. This is fundamental, and very important for students to think about *imaginary* (and *complex* numbers), matrices, and other topics that they will eventually encounter in high schools. Furthermore, this approach will reinforce the logical thinking in mathematics.
6	6.EE.1-6.EE.7	This algebraic method of thinking should not be a problem if students are exposed to teaching mathematics by graphical means as done by *Singapore's* method of teaching mathematics. They encounter this approach early in the elementary school (primary school) years.
6	6.G.1	Finding the area of triangle, trapezoid or other shapes is suitable in the 5^{th} grade. It is directly related to fractions. Teaching this topic in 6^{th} grade takes away valuable time from other toics.
6	General Comments	The main purpose of 6^{th} grade should integrate previously learned materials to solve complex problems. In this regards, students should learn about error analyses (for science), how to measure height of a flag pole (which exists in every school) based on shadows & similar triangles, estimate area of a roof of a home using Pythagoras theorem. Students should experimentally determine π (as a ratio of circumference to the diameter of a circle). These measurements will lead them to better understand science when it comes to laboratory tasks.

6	General Comments	Missing item here is how to find a square root of an arbitrary positive number. This approach is 6th grade material when using graphical approach. We know that this item is not part of standard textbook, but it should be taught as it is easy to derive it. (Students should not have to wait until calculus/Taylor's series to learn how to determine square root of a positive number, or use logarithms or pocket calculators to retrieve the numbers.)
7	7.RP.1	Why repeat this topic in grade 7 if it was covered in depth in 6th grade?
7	7.NS.1	This topic should have been handled earlier in grade 3 (for addition & subtraction of a positive & negative numbers), and grade 4 (for multiplications & division of positive & negative numbers).
7	7.NS.2.a and 7.NS.2.b	The topic of multiplication and division of signed numbers need to be handled in terms of the rules of multiplications and division. Multiplication is repeated addition, and division is grouping. This approach gives information on the mixed numbers. The way to derive $-1\times-1=1$, or $-1\div-1=1$ need to be related to inverse and identity operation, and take advantage of all other facts known. Thus, students should not be taught that this is an arbitrary rule that has been imposed on students. Once this item has been derived, then teach the students the rules (i.e., summary) of mixed number operation. This approach to teaching multiplication will play a major role when students learn complex numbers and matrix algebra (through mathematical operation of matrices).
7	7.G.4	The topic of ratio of the circumference to the diameter of a circle is appropriate in 6th grade. After students learn fraction & expressing them in decimal, then determination of π is one such application. Students should determine this through experiments, instead of believing teachers. [In mathematics, students should not be taught that *I am correct because I am the teacher*, but rather should determine it by themselves. For this, an experimental measurement is required to convince students that π is approximately 3.14 (or 22/7).]
7	General Comments	An important topic that should be taught here is not listed: Prime Numbers. Students should have had enough practice with factors to learn primer numbers fast. A significant number of major topics in 7th grade appears to be repeated material that students may have learned previously. The topics on the Ratios and Proportional Relationships, the Number System, and Expressions & Equations do not appear to be new. The need to pursue these topics is based on how deep these topics went into in previous grades. Repeating these topics prevents students from accessing higher levels materials that they can take vigorous mathematics and science courses, which includes taking calculus in 11th grade.
8	8.NS.1-8.NS-2	This material should be taught in 6th grade. This would be illustrated come about by computing $\sqrt{2}$ by iterative procedure. This can be easily derived by use of graphical approach to teaching mathematic.

8	8.G.6	Proof of Pythagoras (or Pythagorean) theorem should have been done in 5th grade when students learn the area of triangles, and relate them to the area of a square. Furthermore, this topic requires efficient method to compute square of a number (e.g., 35×35) and a square root of a number. Students should not rely on a calculator to compute these quantities. Thus, it would be better if the prerequisite topics of squaring of a number efficiently (by mental math) and finding a square root of a positive number are taught.
8	8.G.7	Application of Pythagoras theorem to real world problems is appropriate in 5th and 6th grade.
HS Num & Quant	General Comments	The missing item here is teaching students how to sum sequences of numbers. In the elementary school years, students have mastered how to add simple sequences of numbers (i.e., Σn). Students should be taught how to add Σn^2, Σn^3, Σn^4, … This can be taught from the first principles. [*This topic may also be part of algebra.*]
HS Algb	A-APR.5	Binomial theorem is very much related to multiplication of a number by ×11. There should be a connection made between ×11 and binomial expansion. Multiplication trick on a number ×11 should have been taught earlier in 3rd grade. Students should have been exposed to binomial theorem in terms of area of a square in 3rd grade. In 5th grade, they should have been exposed to quick way of multiplying mixed fractions. Thus, this should not be the first time when students are exposed to binomial theorem. This will again be utilized in deriving equation for Σn^m, where both n and m are positive integers.
HS Funct	F-BF	There is no explicit mention of parametric equations (i.e., time dependence on spatial coordinates). Students need to know how to solve this problem as it is encountered extensively in physics.
HS Funct	F-TF	Trigonometry should be taught when the angles are purely imaginary. This topic has real world application to special theory of relativity. This approach to teaching special theory of relativity is simple. It teaches students about the fundamental definition of trigonometric, and determine if the identities survive when the angles become pure imaginary. This is creative application of complex number in trigonometry. Students should be exposed to different coordinate systems, i.e., Cartesian, polar and spherical systems. This is very much related to finding an area of a surface, or vectors, etc. [This may belong to function.]
HS Geom	Geometry Gen Cmnt	There is no explicit discussion of solid angles. This fact is important for those students planning to take physics.
HS	General Comments	For STEM curriculum, students need to know about linear and matrix algebra. This includes finding eigenvalues and eigenvectors of a symmetric (or Hermitian matrix), unitary transformation, vectors, how to find determinant, etc. Matrices play important role in STEM, and will make it easy for students to solve problems in physics.

Date: April 21, 2016
Name: **Suketu Gandhi**

Signature:

City: **Idaho Falls**
State: **Idaho**
Stakeholder Group: **Parent**

Reference: Common Core Mathematics Standards Revision

Mathematics KG (Kindergarten)

Measurement and Data: Students should know the time. The example is when sun rises, when people go to bed at night, when the school starts & finishes, etc. 12 hour (hour hand/minute hand) clock falls into the category of number ≤ 20.

Students should know the calendar (i.e., days of the week, number of days in a month, and how to designate year). Days in a month, and number of months in a year are related. This is related to teaching numbers to students.

Students should be taught temperature to know the number that corresponds to hot & cold.

Number: Why students should count to 100, but not write to, say 50? They need to learn to read.

Money: Students should know the money as in dollars and cents and alternate expressions (e.g., quarters, dimes, nickels and pennies), and learn how to add them.

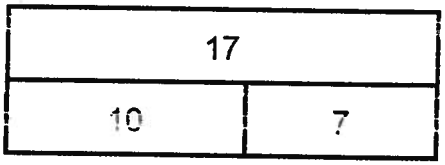
Geometry, *Shapes, their attributes, and spatial reasoning*:
1) Include directions such as left & right in mathematics. It will be used over & over again to describe many mathematical situation, such as position in a line, etc.

Mathematics Grade 1

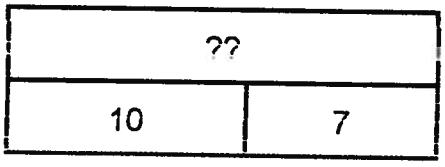
General Comments: Number system does not end at 20. Children already know the numbers up to 50 from KG. Children should know addition & subtractions of the numbers upto 100. However, teachers should teach materials in such a way that starts with addition & subtraction operation upto 20 early, and then gradually progress to 100 by the end of the school year.

Addition and Subtraction: Limiting addition and subtraction to 20 stagnates student's mind, and keeps standard very low. Students should learn how to add and subtract numbers involving sums upto 100.

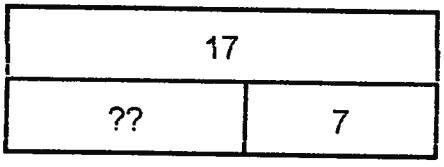
Introduce graphical approach to addition and subtraction. It will help students to better understand the relationship between addition and subtraction.



Consider *10+7=17* in graphical approach. Addition is putting together two numbers *10+7*.



Graphical approach to subtraction is illustrated similarly. This forms intuition on the relationship between addition and It would help students who may not be proficient with words, but understand the content.



The approach described in the draft does not provide students to understand the concept through illustration. Furthermore, the students should be taught how to add numbers with carrying, and subtract numbers with borrowing.

Measurements and Data: Please discuss concept of weight. This is extremely important for comparing numbers, just as is the case for length measurements. Illustrative example can be given such as the weight of two objects, etc. Concept of weight measurement should be also done with illustration, as done for addition. One can ask students the when the weights are equal, or not equal. This topic reinforces comparison between numbers and connects them to understanding of weight. There are many thinking problems based on weight. It is hoped that concept of pound/kilogram would be introduced.

It is hoped that teachers would make primitive scale in class (as part of art & craft). This will be easy to connect with weight. Two separate methods of measurements (pounds/ounces, and kilogram/gram), they should be taught as separate language. Then students don't translate from one unit to another units.

Money: Students should know how to add money as in quarters, dimes, nickels and pennies.

Geometry: Topic not included here is the measurements of perimeter certain shapes (e.g., squares, triangles and rectangles.). The measurements reinforce the concept of addition. Topic of symmetry (i.e., drawing the line of symmetry, for mirror symmetry) should be included in here.

Fractions: Students should be exposed to expression of fraction. In time, students should know what is meant by ¼ hour, ½ hour, etc. Fraction is difficult topic to master, but students should be exposed to it. Addition and subtraction of fraction is easily handled by eating *pizza* slices.

Temperature: Temperature should be measured to illustrate what is cold and hot.

Mathematics Grade 2

Addition and Subtraction: Students should know how to do addition and subtraction with carrying and borrowing. It is hoped that students have mastered this concept in the 1st grade. This should be reinforced. As discussed in **Mathematics Grade 1**, use illustration to describe addition and subtraction. Students should practice this with at least three digit numbers.

Multiplications: This topic must be taught in grade 2. Multiplication is extension of repeated addition. Students should know their multiplication tables from $\times 1$–$\times 10$. It is not clear as to why this should not be taught to the students.

Numbers: Students should be taught negative numbers. This would be natural extension of subtraction. The concept behind this topic can be taught by making analogy with elevator going up and down a building. When introducing this topic, numbers should be restricted to ± 50. Students should be taught the meaning of a very large number or infinity (∞).

Fractions: Students should be exposed to fraction. They should continue to add and subtract fraction with the same denominator using graphical approach (i.e., pizza pie). They can compare fraction (by working with art & craft). Money can be related to fractional amount (i.e., ¼ of dollar is quarter = 25 cents).

Measurements: Please include measurements of area. Weight and temperature measurements were discussed in **Mathematics Grade 1**. Discussions of temperature should include freezing and boiling point of water. Further discussion should be made as to temperature below freezing (i.e., negative temperature). In weight, students should go into ounces, and grams. This topic should be connection with fraction. Time should be taught every 5 minutes, and connection can be made with learning of multiplication table.

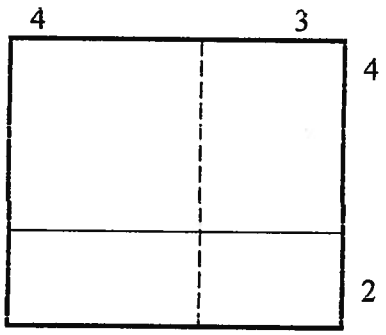
Money can be used in problems, but not as a new topic. Students should have mastered this topic, and be able to utilize money through word problems.

Geometry: Students should be able to measure and compute perimeter of various geometric objects (e.g., triangle, squares, etc.). Topic of symmetry should touch rotational symmetry. This will be important to lead student into finding area of triangle.

Mathematics Grade 3

Students should know their multiplication tables from ×1–×10 when they enter 3rd grade. They should have completed their addition and subtraction of whole number by this time. (They should be proficient in addition with carrying, and subtraction with borrowing.) They should be proficient with adding money from the 2nd grade. By this time, students should know how to express fraction.

Multiplications and Division:
Multiplications: From the 2nd grade, students should know the area, and relate area graphically. When multiplication is connected with graph, it will be easy for them to visualize. Teaching of distributive property of multiplication should be illustrated. This way, students learn materials without learning complicated words. The figure on the left demonstrates this. The area of the rectangular object is
$7\times6 = (4+3)\times(4+2) = 4\times4 + 3\times4 + 4\times2 + 3\times2$.



3rd grade, students should learn their multiplication tables ×11–×20. They should be taught how to carryout multiplication of two digit numbers by both single and double digit numbers.

Division: Students should learn how to divide whole numbers in one, two and three digit by a single digit number. They should learn how to express remainder (e.g., 30÷7=4, 2Remainder)

Mental Math Tricks: Students should know simple ways to multiply two digit numbers involving ×11, squaring of a number with 5 in the ones digit (e.g., 75×75)/squaring of any two digit number. Mental math tricks should be taught on how to multiply a number by 2, 4 and 5. The mental multiplication tricks will help students solve problem quickly.

Quick computation of a number ×11 is illustrated on the right. This is well known, and it can be easily derived. This trick will help out students in later years when they take their course in Algebra to study Pascal's triangle.

a	b	
× 1	1	
a	b	
a	b	0
a	$(a+b)$	b

a	b	c	
×	1	1	
a	b	c	
a	b	c	0
a	$(a+b)$	$(b+c)$	c

Multiplication of a number ×2 is easy. It requires doubling. Multiplication of a number ×4 is easy, it involves doubling of a number, and then redoubling it again. Multiplication of a number by ×5 is quick: First, multiplying a number ×10, and then this number dividing by 2.

Squaring of a number is easy, and it should be taught to students. For example given a number, N, and one wishes to find N^2. $N^2 = (N+m)\times(N-m) + m^2$. Let us choose, N=35, and $m=5$:

$$35\times35 = (35+5)\times(35-5) + 5\times5 = 40\times30+25 = 1200+25 = 1225.$$

m is chosen so that the first term (i.e., N+m) ends in zero. Under this circumstance, it is easy to multiply a two digit number by a one digit number (or a two digit number having 0 in ones position). This is illustrated in 68×68. We choose $m=2$, and this is illustrated as follows:

$$(68+2)\times(68-2) + 2\times2 = 70\times66 + 4 = 4200+420 + 4 = 4624.$$

Students should be taught how to convert problem of addition to that involving multiplication (as done by Carl Friedrich Gauss) in adding sequences of numbers. Again, graphical approach to addition (via power point) presentation would be beneficial to students. One can start teaching students by adding up sequences of numbers of 1, 2,…10, and later it can be extended to a larger number.

Fractions: Multiplications of fraction should be introduced as repeated addition. They should learn how to get a whole number from fraction multiplication. The topic should be restricted to a whole number multiplied by a fraction. Students can connect the materials learned in 2nd grade with fraction in 3rd grade.

Mathematics Grade 4

Students should enter 4th grade with full knowledge of addition & subtraction with carrying & borrowing, multiplication of a two digit number by a two, and a three digit numbers. Thus, multiplication of whole number should be closed (i.e., completed by this time). They should know how to divide a two and three digit number by one digit number, and addition & subtraction of positive and negative numbers.

Students should master how to divide a number by a two digit number, and learn how to express remainder. They should learn how to factor numbers to make their division easier.

Division: Students should learn how to divide a number by two or more digit number. Students should start learning how to use factors to simplify division.

Numbers: Students should learn how to multiply and divide with both positive and negative numbers.

Geometry: Students should start to measure angles, and determine sum of internal angles of triangle, etc.

Mathematics Grade 5

When students enter 5th grade, they should master every element of fraction: Addition, subtraction, multiplication and division. They should continue with this by learning about decimal, and ratio and percent (%).

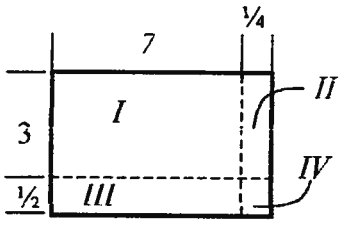
Fraction: Students should learn fraction through graphics (or modeling, illustration, diagram, etc.). For example, division of a whole number by a fraction should be illustrated in its fundamental form. 1/(¼) should be illustrated that one wants to make groups of ¼. Illustration by diagram would show that quotient is 4.

Multiplication of mixed fraction should be done by two separate approach: 1) Traditional, 2) graphical. Traditional approach requires significant amount of effort, but the graphical approach will make computation easy. Graphical approach occurs when a connection between improper fraction multiplications and area of a rectangle.

Example of this is illustrated below for the case of (3½)×(7¼):

$$7\tfrac{1}{4} \times 3\tfrac{1}{2} = \frac{29}{4} \times \frac{7}{2} = \frac{29\times 7}{8}$$

$$= \frac{203}{8} = 25\tfrac{3}{8}\ .$$



By drawing a box, we divide it up into four segments (i.e., *I*, *II*, *III and IV*). They consist of the whole number and the fraction. We note that the total area consists of the four individual parts. Thus,

$$7¼\times3½ = 3\times7 + 3\times¼ + 7\times½ + ¼\times½ = 21\ (I) + ¾\ (II) + \tfrac{7}{2}\ (III) + ⅛\ (IV)$$
$$= 21 + ¾ + 3½ + ⅛\ .$$

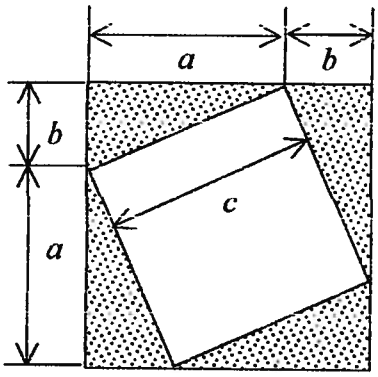
Further simplification gives

$$24 + ¾ + ½ + ⅛, \text{ or } 25\tfrac{3}{8},$$

which we obtained by the previous two methods as illustrated above. The intermediate work requires the drawing of the rectangle and breaking it upon into segments. When these approaches are taught, students can determine the fastest way to determine the correct answer.

Decimal: Students should be fluent in writing decimals with 2,3,4,5,6,8,9,10,12,16 and 100. in the denominator. This is possible for students when they know the times tables from ×1 – ×20. It is not clear why the denominator be restricted to 2, 4, 5, 8, 10 and 100. Students should also learn how to do computation of the above fraction using decimal.

Geometry: Students should be exposed to the meaning of a right triangle (and right angle). Students should learn the area of triangle and trapezoids. By this time, they know how to compute area of a rectangle, and know the lines of rotational symmetry. Thus, students are proficient in understanding in the derivation of equation of area of a triangle and trapezoids. Students should also be taught the **Pythagoras Theorem,** which illustrates multiplications.



This can be can be shown (on the right) by using two squares of whose sides have length $(a+b)$ for the larger one, and c for the smaller one. The area of the large square is the sum of the area of the smaller square and the four triangles, or $(a+b)\times(a+b)$. Thus, the area of the larger square is simply

$$(a\times a) + (b\times b) + (a\times b) + (b\times a) = 4\times\frac{a\times b}{2} + (c\times c)\,.$$

Upon simplification, we get the relationship between the sides of the triangle and its hypotenuse $(a\times a) + (b\times b) = c\times c$. (The notations used are the ones that would be familiar to the students before pre-algebra.) This may be taught in high school geometry, but it is possible to teach the materials shortly after learning areas of squares and triangles.

Students should be given problems on perimeter involving triangles as they know Pythagoras Theorem.

Ratio: Students should know the ratio in context of geometry and fraction, and able to determine the area of a shape given the ratio of linear dimensions.

Percentage: This topic should be taught as a part of decimal. Example of this may be the discount offered in stores (i.e., weekend special, 15% off, early morning special, 15% additional savings, etc.), and compute total amount of discount. One should resort to mental math to do these computations.

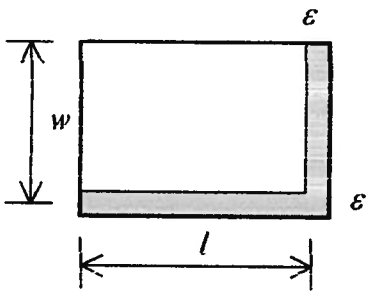
Mathematics Grade 6

By this time, students would have learned all of the materials on addition, subtraction, multiplications, division, fraction, decimals, percentage, ratio, positive & negative numbers. They would have mastered every thing about area, and volume. Here, students should integrate various materials, and set on the trajectory to solve variety of problems that integrates many elements together. Here, students should be taught many elements of pre-Algebra.

Geometry: Students should learn how to measure the height of a flagpole (which is a standard in every public school) via shadows measurements of known object. This would reinforce similar triangles & ratio. Students should be taught about π ('pi') as they know decimal, and division of a number by a decimal. To strengthen student's scientific ability, they should experimentally determine π by measuring the circumference and the diameter of a large object.

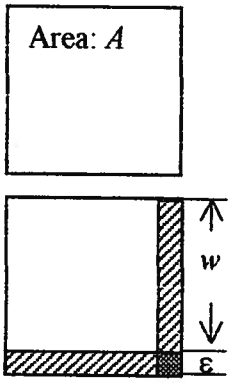
Numbers: Students should learn power (a^n, 10^m), square root, and be able to compute square root of a positive number. Furthermore, the students should be taught about purely imaginary number (i.e., $\sqrt{-1} = i$). They should learn how to express significant digits in measurements. This will be needed to determine height of a flag pole through experimental measurements.

Students should be taught about error analysis through uncertainty in measurements. Graphical approach to error analysis should be related to measurements of an area with uncertainty in the measurements of length. This topic leads to laboratory measurements in physics & chemistry.



Proper question to ask is given uncertainty in length & width measurement of 5%, what is the error in determination of an area. Both positive and negative numbers would be needed. In this case area is expressed as

$A = (l \pm \varepsilon) \times (w \pm \varepsilon) = l \times w \pm (2 \times \varepsilon)$, where we neglect terms in ε^2.
Note: Rigorous error analysis (i.e., root mean square of all errors divided by number of variables-1).



Square Root of a Positive Number: The present approach in teaching mathematics does not include how to find (or determine) a square root of an arbitrary positive number. Students may learn in calculus [when taking AP Calculus (BC), where Taylor's series is part of the curriculum. However, this topic is not part of Calculus AB.] In schools, students are taught how to retrieve a square root of a number (i.e., use of calculator, or logarithm tables), which is not the same as the determining from the first principles.

However, the graphical approach significantly helps to solve the problem through instilling in the intuition, and makes it accessible to students early. This is clearly illustrated in determining square root of an arbitrary positive number. It is easy to grasp. The first thing to note is that finding a square root of a number is equivalent to finding the length of a square given its area. This topic should be suitable for those learning pre-Algebra, as it may be the case for the 6th grade students. (By this time, the students should must know both the positive and negative numbers and be fluent with fractions.)

We note that the square root of the area (A) is sum of the length ($w+\varepsilon$), as shown below figure on the left. Thus, one can easily express

$$A = (w + \varepsilon)^2 = w^2 + (2 \times w \times \varepsilon) + \varepsilon^2 \quad .$$

This is a graphical approach to deriving binomial expansion. (Binomial expansion considered part of Algebra). We note that as ε becomes smaller (i.e., $\varepsilon \to 0$), we approximate Area by neglecting ε^2, as done in Eq.(1)

$$A = w^2 + (2 \times w \times \varepsilon) \quad . \tag{1}$$

One solves for ε, and gets the expression

$$\varepsilon = \frac{A - w^2}{2 \times w} \quad . \tag{2}$$

One estimates the new w as the old $w+\varepsilon$ through iteration (or repeating the process again). This leads to decrease in the ε. As ε becomes smaller, the approximated area resembles the true area. Neglect of the small area ε^2 means neglecting a very small number. As a result, this determines numerically the square root of a number, instead of retrieving it from a table.

To better illustrate this approach, let us determine $\sqrt{2}$. We approximate $\sqrt{2}$ as 1.5, since $1.5 \times 1.5 = 2.25$ (which is known from the mental math). Using this in Eq.(2), we get $\varepsilon = (2-2.25)/(2 \times 1.5)$

= -0.25/3.0 = -1/12 = -0.083333. As a result, the new w is 1.5-0.0833333, or 1.41666666. We note that $(1.416666)^2 = 2.0069444$. (The exact value of $\sqrt{2}$ is 1.4142135.) The following table illustrates iterative approach to finding $\sqrt{2}$.

No	*Area*	*w*	w^2	ε
1 (initial)	2	1.5	2.25	-0.08333333
2	2	1.41666666	2.00694444	-0.00245098
3	2	1.414215686	2.00000600	0.00000021

Cube Root of a Number: Similar approach (as shown above for finding square root) can be taken to find the cube root of a positive number. Teaching this topic integrates binomial expansion [i.e., $(w+\varepsilon)^3$], teaches students more about how to think creatively, and solve difficult problems easy way. Solving this problem without calculator will help students reinforce topics on addition/subtraction/multiplications/division, fraction, positive & negative numbers. This will force students to think in finding exact solutions numerically. It would complement their finding cube of a number.

Mathematics Grade 7

Students should be ready to enter Algebra class by this time. The topics described in here should be taught in 6th grade. The missing topic here is imaginary number (i.e., $\sqrt{-1} = i$).

Mathematics Grade 8

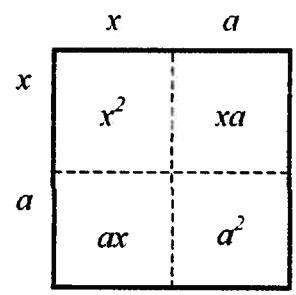
Students should learn (i.e., $\sqrt{-1} = i$), if they have not learned it already.

Mathematics High School Algebra

Complex Number System: First major problem in the complex number (or imaginary number) is the vocabulary. Students must not confuse complex number as "complicated" numbers, or imaginary number as "fantasy" numbers. It should be introduced in abstract way with a fictional plausible story: *Mathematical rules are invented to solve specific problem. In the days of caveman, the counting was invented to determine number of sheep. A few thousand years after the cavemen invented counting on hands, they learned how to count beyond ten, by repeating every ten. After that the mathematician of the time invented multiplication as repeated addition. A few generation after that, a person asked very simple question: ?+5 =0. To solve this problem, negative numbers were invented. Anther 150 years later, another person asked ?×5 = 1. This lead to invention of fractions. The student of this person asked another question, if we have 100 corn, how do we give everyone in the tribe equal amount of corn? This lead to the invention of division. Now in modern times, mathematician asked a question what number × what number gives –1 (i.e., ?×? = -1)? Solution to this problem is called an imaginary number, as the numbers we know it does not solve the problem.* This fictional story leads to idea that complex number are solution to mathematical equation. So that students accept this material.

Also give concrete examples of meaning of commutative property. For example, one can use analogy with washing clothes. First part is to wash clothes, the second part is to dry these clothes. Reversing such operation does not give the same results (i.e., dry clothes, then wash them).

Arithmetic with Polynomials and Rational Expressions: Binomial theorem. This should be taught both graphically and normally (i.e., Pascal's triangle). Relate this to multiplication of ×11. Graphical approach to binomial theorem is easy to understand, as students would have came across this topic in 6th grade (as per earlier comments).



Important component missing from the binomial expansion is how to use it properly for a number of things. This has been pointed out in finding square root, and a cube root of a number. It should be used to aid in error analysis. It should be used to find a compact expression (or formula) for adding series of numbers (which is different from the Carl Friedrich Gauss's approach, which the students should have mastered when they learn multiplications of two digit numbers by another two digit numbers).

Consider summation of a series of number: Σn (i.e., 1+2+3+, ...). To find compact equation, one has to do summation for following sequences of number,

$$\Sigma(n+1)^2-n^2 = 2^2-1^2+3^2-2^2+4^2-3^2+\ldots$$

$$= \Sigma(n^2+2n+1-n^2) = \Sigma(2n+1)$$

Compact expression is derived by defining both the first and the last number in the sequence (i.e., M), and solve for the expression. The surviving terms are $(n+1)^2$ for $n=M$, and n^2 for $n=1$. We get

$$\sum_{n=1}^{M}(n+1)^2-n^2 = \sum_{n=1}^{M}2n+1 = \left(\sum_{n=1}^{M}2n\right)+M = (M+1)^2-1.$$

After algebra, we get

$$\sum_{n=1}^{M}n = \frac{M^2+M}{2} = \frac{M(M+1)}{2}$$

This provides the compact expression of Σn.

To obtain Σn^2, we solve for $\Sigma[(n+1)^3-n^3]$. By binomial expansion, we get

$$\Sigma(n+1)^3-n^3 = \Sigma n^3+3n^2+3n+1-n^3 = \Sigma 3n^2+3n+1.$$

We note that on the right hand side of the equation, the surviving terms are known once the initials and final numbers of the sequences are defined. Let us consider sequences of numbers starting from 1, and ending at M, as done before.

$$\sum_{n=1}^{M}(n+1)^3-n^3 = \sum_{n=1}^{M}3n^2+3n+1 = (M+1)^3-1.$$

The surviving terms are the first one corresponding to n^3 and the last one corresponding to $(M+1)^3$. All other terms disappear due to addition and subtraction. By further simplification, we get following

$$\sum_{n=1}^{M} 3n^2+3n+1 = \sum_{n=1}^{M} 3n^2 + \left(\sum_{n=1}^{M} 3n\right) + \left(\sum_{n=1}^{M} 1\right) = (M+1)^3-1.$$

This is simply known from the above results as $3M(M+1)/2$ ↑ M ↑

This gives expression for Σn^2 that can be easily solved by conducting algebra.

The same approach can be used to obtain Σn^k, where k is an integer >1. Applications of binomial expansion reinforce the previously learned materials, and teach students how to add sequences of numbers from the first principle. This is possible when learning Algebra. (This is alternate approach than mathematical induction, which does not teach on systematic approach to solve this type of problem.)

Mathematics High School—Functions

Trigonometric Function: Include a topic on trigonometry when angles are complex (or pure imaginary). This will make it easy for students to learn Special Theory of Relativity (a modern physics), which may be taught in AP Physics (C) class.

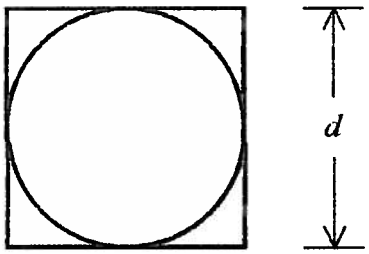
Furthermore, there is no explicit mention of connecting trigonometry with vectors (e.g., parametric equations). This is needed for students taking full year of physics class in high school. Students should be given equation in terms of exponents [i.e., $\exp(i\theta)=\cos\theta + i\sin\theta$)] to help solve trigonometric identifies. This makes it easy to solve various identities.

Students should plot function on polar coordinates. This has not been mentioned explicitly for trigonometry.

Modeling and Geometry: Should not use BTU. In science & engineering, everything is metric. Students should look at density in terms of metric quantity. Also, students should be exposed to different expression of density (e.g., surface density, energy density, etc.)

Mathematics High School—Numbers and Quantity

Vector Quantities and Matrices: Many topics are not mentioned with respect to matrix algebra. Matrices are important in a number of ways. There is an explicit mention to help solve system of equations, but there is no mention of aids in vectors to compute both the cross (vector product) and the dot (inner) products in specific terms. It helps students in transforming coordinate system through unitary transformation, finding characteristic equation (i.e., eigenvalue equation). Students should know how to obtain determinant for both 2×2 and 3×3 matrix. Students should relate 3 dimensional spherical coordinates with matrices. (They can use this approach to learn Special Theory of Relativity.)

Mathematics High School—Statistics and Probability

Probability: This topic is very important in terms learning to solve certain mathematical problems. However, no mention has been made on making connection with previously learned materials. For example, when students learn about random numbers with uniform distributions, they can use it to determine "pi" (π). For example, random numbers distributed uniformly within a square, one can determine ratio of the random number lying within the circle of diameter d to the square of length d. This would give better understanding on random number.